

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\frac{1}{2}e^{2x+1} + C$

2.2

- $\frac{1}{2} \ln |2x + 3| + C$

2.3

- The numerator is the derivative of the denominator; $= \ln |3x^2 + 1| + C$

2.4

- $-\frac{1}{3} \cos(3x - 1) + C$

2.5

- $\int \frac{1}{2}(1 + \cos 2x) dx = \frac{1}{2}x + \frac{1}{4} \sin 2x + C$

3.1 B

- $\int \cos 4x dx = \frac{1}{4} \sin 4x + C$

3.2

- $4 \cos^2 x = 2(1 + \cos 2x); \int_0^{\pi/4} (2 + 2 \cos 2x) dx = [2x + \sin 2x]_0^{\pi/4}; = (\frac{\pi}{2} + 1) - 0 = \frac{\pi}{2} + 1$

3.3

- $h = 0.5; y$ at $x = 0, 0.5, 1, 1.5, 2$: $1, \sqrt{1.125} = 1.0607, \sqrt{2} = 1.4142, \sqrt{4.375} = 2.0917, \sqrt{9} = 3; \int \approx \frac{0.5}{2}[1 + 3 + 2(1.0607 + 1.4142 + 2.0917)]; = 0.25[4 + 2(4.5666)] = 0.25(13.133) = 3.28$ (3 s.f.)

3.4

(a)

- Take $u = \ln x$, because it differentiates to the simpler $\frac{1}{x}$, and $\frac{dv}{dx} = x$ so $v = \frac{1}{2}x^2$;
 $= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x dx; = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

(b)

- $u = 1 + x^2$ so $\frac{du}{dx} = 2x$ and $x dx = \frac{1}{2} du$; the limits become $u = 1$ and $u = 5$;
 $\int_1^5 \frac{1}{2}u^{-1/2} du = [u^{1/2}]_1^5; = \sqrt{5} - 1$

(c)

- $\frac{5x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$, so $5x+1 = A(x-1) + B(x+2)$; $x = 1$ gives $B = 2$,
 $x = -2$ gives $A = 3$; $\int \frac{3}{x+2} + \frac{2}{x-1} dx; = 3 \ln |x+2| + 2 \ln |x-1| + C$