Solutions

Each answer shows the steps, then the **result**.

2.1

- $\frac{1}{2}e^{2x+1} + C$

2.2

- $\frac{1}{2}\ln|2x+3| + C$

2.3

- The numerator is the derivative of the denominator; $= \ln|3x^2+1| + C$

2.4

- $-\frac{1}{3}\cos(3x-1) + C$

2.5

- $\displaystyle\int \tfrac{1}{2}(1+\cos 2x)\,dx = \tfrac{1}{2}x + \tfrac{1}{4}\sin 2x + C$

3.1 B

- $\int \cos 4x\,dx = \frac{1}{4}\sin 4x + C$

3.2

- $4\cos^2 x = 2(1+\cos 2x)$; $\displaystyle\int_0^{\pi/4}(2+2\cos 2x)\,dx = [2x+\sin 2x]_0^{\pi/4}$; $= \left(\frac{\pi}{2}+1\right) - 0 = \frac{\pi}{2}+1$

3.3

- $h = 0.5$; y at $x = 0, 0.5, 1, 1.5, 2$: 1, $\sqrt{1.125} = 1.0607$, $\sqrt{2} = 1.4142$, $\sqrt{4.375} = 2.0917$, $\sqrt{9} = 3$; $\int \approx \frac{0.5}{2}[1+3+2(1.0607+1.4142+2.0917)]$; $= 0.25[4+2(4.5666)] = 0.25(13.133) = 3.28$ (3 s.f.)

3.4

(a)

- Take $u = \ln x$, because it differentiates to the simpler $\frac{1}{x}$, and $\dfrac{dv}{dx} = x$ so $v = \frac{1}{2}x^2$; $= \frac{1}{2}x^2\ln x - \displaystyle\int \tfrac{1}{2}x\,dx$; $= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$

(b)

- $u = 1 + x^2$ so $\dfrac{du}{dx} = 2x$ and $x\,dx = \frac{1}{2}\,du$; the limits become $u = 1$ and $u = 5$; $\displaystyle\int_1^5 \tfrac{1}{2}u^{-1/2}\,du = \left[u^{1/2}\right]_1^5$; $= \sqrt{5} - 1$

(c)

- $\dfrac{5x+1}{(x+2)(x-1)} = \dfrac{A}{x+2} + \dfrac{B}{x-1}$, so $5x+1 = A(x-1) + B(x+2)$; $x = 1$ gives $B = 2$, $x = -2$ gives $A = 3$; $\displaystyle\int \frac{3}{x+2} + \frac{2}{x-1}\,dx$; $= 3\ln|x+2| + 2\ln|x-1| + C$