

2.4 Differentiation (product, quotient, parametric, implicit)

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS parametric 参数 • parametric equations 参数方程 • product 乘积法则
• implicit 隐式 • quotient rule 商法则

Objective

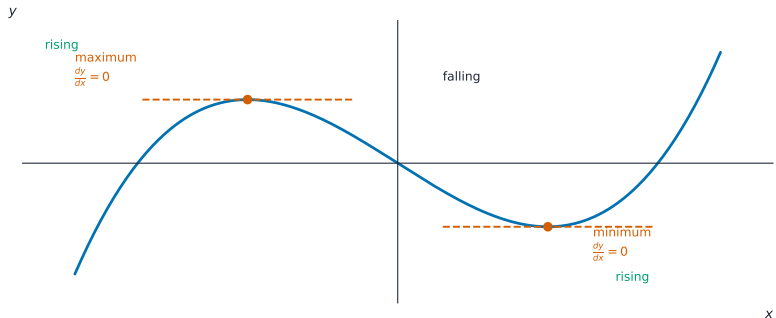
Build the skills to answer exam questions on **differentiation** 微分 — standard derivatives (e^x , $\ln x$, trig), the **product** 乘积法则 and **quotient rules** 商法则, and **parametric** 参数 and **implicit** 隐式 differentiation.

You must be able to:

- differentiate e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$
- apply the product and quotient rules
- differentiate parametric and implicit relations

1 Worked examples

■ Product rule & implicit



stationary points: $\frac{dy}{dx} = 0$ · use $\frac{d^2y}{dx^2}$ or first-derivative sign to classify

A stationary point ($\frac{dy}{dx} = 0$) is still found by differentiating — now with the product/quotient/chain rules

$y = 6x \cos(x^2 + 1)$. Product rule ($u = 6x$, $v = \cos(x^2 + 1)$):

$$\frac{dy}{dx} = 6 \cos(x^2 + 1) - 12x^2 \sin(x^2 + 1).$$

Implicit: for $x^2 + y^2 = 25$, differentiate: $2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$.

2 Practice

2.1 Differentiate $y = x^2 e^x$. [2]

2.2 Differentiate $y = \frac{\sin x}{x}$. [2]

3 Exam-style questions

3.1 $\frac{d}{dx}(\ln(3x))$ equals: [1]

- | | |
|---|---|
| A | B |
| C | D |
- A $\frac{1}{3x}$
B $\frac{3}{x}$
C $\frac{1}{x}$
D $3 \ln x$

3.2 A curve has parametric equations $x = t^2$, $y = t^3 - 3t$. Find $\frac{dy}{dx}$ in terms of t , and the value of t at the stationary point(s). [4]

3.3 A curve is defined implicitly by $x^2 + xy + y^2 = 7$.

(a) Find $\frac{dy}{dx}$ in terms of x and y . [3]

(b) Find the gradient of the curve at the point $(1, 2)$. [2]
