2.1 Algebra (modulus, factor and remainder theorems)

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS factor 因式定理 • remainder 余数 • remainder theorem 余数定理 • polynomial 多项式 • modulus 绝对值

Objective

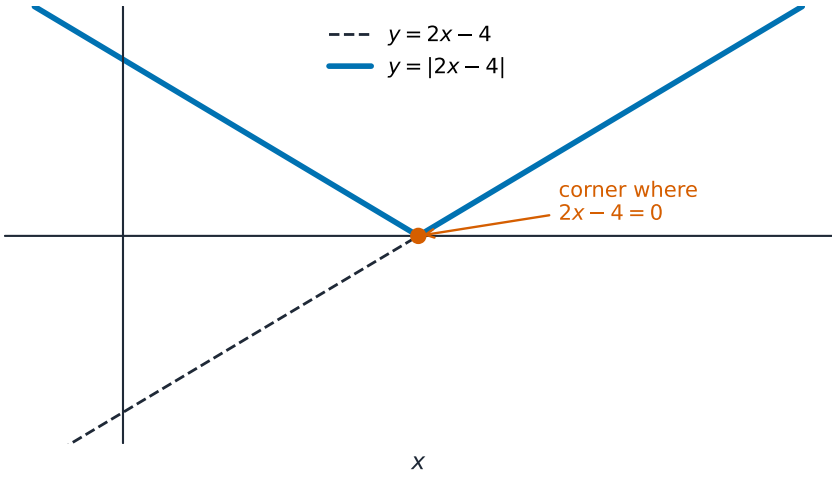
Build the skills to answer exam questions on **algebra** 代数 — the **modulus** 绝对值, and the **factor** 因式定理 and **remainder theorems** 余数定理 for polynomials.

You must be able to:

- solve modulus equations and inequalities ($|x-a|<b$, $|a|=|b| \Leftrightarrow a^2=b^2$)
- use the remainder theorem (remainder $= p(a)$)
- use the factor theorem to factorise a polynomial

1 Worked examples

■ Modulus & factor theorem



$y=|f(x)|$ reflects parts of $y=f(x)$ below the x-axis above it

Taking the modulus folds the part below the axis up into a "V"

Solve $|3x+8|<9$. $-9<3x+8<9 \Rightarrow -17<3x<1 \Rightarrow -\frac{17}{3}<x<\frac{1}{3}$.

> $p(x) = 2x^4 + kx^3 + kx^2 + 17x + 18$ **has factor** $(x + 2)$**. Find** k**.** $p(-2) = 0$: $32 - 8k + 4k - 34 + 18 = 0 \Rightarrow 16 - 4k = 0 \Rightarrow k = 4$.

2 Practice

2.1 Solve $|2x - 1| = 7$. [2]

2.2 Find the remainder when $x^3 - 2x + 5$ is divided by $(x - 2)$. [2]

3 Exam-style questions

3.1 $(x - 3)$ is a factor of $x^3 - 4x^2 + x + c$. The value of c is: [1]

A B
C D

A -6
B 6
C 12
D -12

3.2 Solve the inequality $|x + 2| > |x - 4|$. [3]

3.3 The polynomial $p(x) = x^3 + ax^2 + bx + 6$ has a factor $(x - 1)$ and leaves a remainder of 8 when divided by $(x + 1)$.

(a) Form two equations in a and b and solve them. [5]

(b) Hence factorise $p(x)$ completely. [3]