

1.7 Differentiation

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS stationary point 驻点 • normal 法线 • chain rule 链式法则
• tangents and normals 切线与法线 • second derivative 二阶导数

Objective

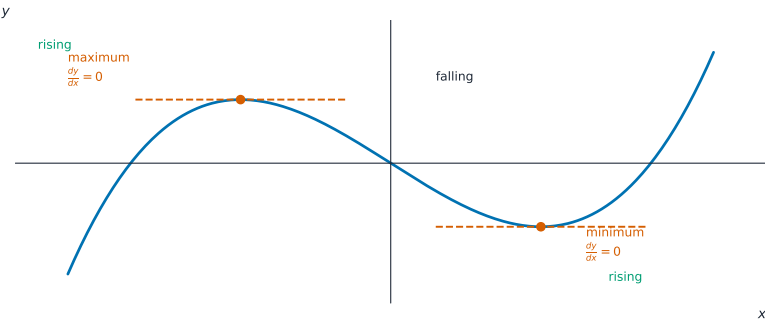
Build the skills to answer exam questions on **differentiation** 微分 — the power and **chain rule** 链式法则, **tangents and normals** 切线与法线, increasing/decreasing, and **stationary points** 驻点.

You must be able to:

- differentiate powers and use the chain rule
- find the equations of a tangent and a normal
- find and classify stationary points (max/min) with $f''(x)$

1 Worked examples

■ Stationary points



stationary points: $\frac{dy}{dx} = 0$ · use $\frac{d^2y}{dx^2}$ or first-derivative sign to classify

At a stationary point $\frac{dy}{dx} = 0$; $f''(x) > 0 \rightarrow \text{minimum}$, $f''(x) < 0 \rightarrow \text{maximum}$

$y = 4x^{1/2} - x$ has a maximum at $x = a$. Find a .

$$\frac{dy}{dx} = 2x^{-1/2} - 1 = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4, \text{ so } a = 4.$$

Chain rule: $\frac{d}{dx}(2x - 1)^4 = 4(2x - 1)^3 \cdot 2 = 8(2x - 1)^3$.

2 Practice

2.1 Differentiate $y = 3x^4 - \frac{2}{x}$. [2]

2.2 Find $\frac{dy}{dx}$ when $y = (3x + 1)^5$. [2]

3 Exam-style questions

3.1 The gradient of $y = x^3 - 2x$ at $x = 2$ is: [1]

- | | |
|---|---|
| A | B |
| C | D |
- A 4
B 6
C 10
D 12

3.2 A curve has equation $y = x^3 - 3x^2 - 9x + 5$.

(a) Find the coordinates of the two stationary points. [4]

(b) Use the second derivative to determine the nature of each, and hence write down the set of values of x for which the curve is decreasing. [3]

3.3 The curve $y = \frac{8}{x}$ passes through $P(2, 4)$. Find the equation of the **normal** to the curve at P . [4]
