

6.5 Hypothesis tests

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS significance level 显著性水平 • type ii error 第二类错误 • type i error 第一类错误
• rejection region 拒绝域 • two-tailed 双尾

Objective

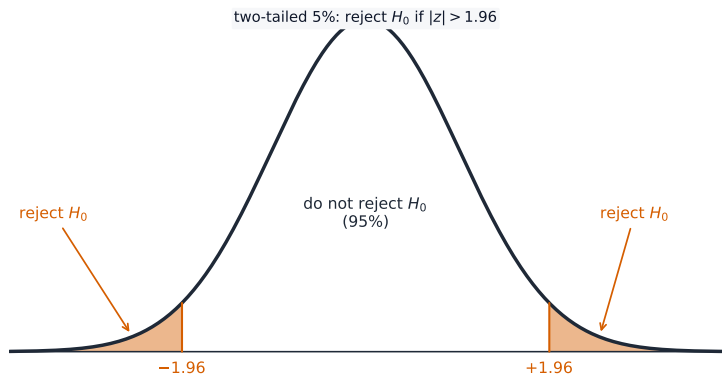
Build the skills to answer exam questions on **hypothesis tests** 假设检验 — stating H_0 and H_1 , one- and two-tailed tests, the **test statistic** 检验统计量 and **rejection region** 拒绝域, and **Type I/II errors** 第一/二类错误.

You must be able to:

- state H_0 , H_1 and choose one- or two-tailed
- compute a z test statistic and compare with the critical value
- explain Type I and Type II errors

1 Worked examples

■ A two-tailed test



A two-tailed test at 5% rejects H_0 only if the statistic falls beyond ± 1.96

Claim $\mu = 50$, $\sigma = 8$, $n = 64$, $\bar{x} = 52$, **test at 5%:** $H_0 : \mu = 50$, $H_1 : \mu \neq 50$.

$$z = \frac{52 - 50}{8/\sqrt{64}} = 2. \text{ Since } 2 > 1.96, \text{ reject } H_0.$$

2 Practice

2.1 State the meaning of a **Type I error**. [1]

2.2 Give the critical z value for a one-tailed test at the 5% level. [1]

3 Exam-style questions

3.1 In a test at the 5% level, H_0 is rejected when the p -value is: [1]

- **A** greater than 0.05
- **B** less than 0.05
- **C** equal to 1
- **D** negative

3.2 A machine should fill bottles to a mean of 500 ml with $\sigma = 6$ ml. A sample of 36 bottles has mean $\bar{x} = 502$ ml. Test at the 5% level whether the mean has increased.

(a) State H_0 and H_1 , and whether the test is one- or two-tailed. [2]

(b) Carry out the test and state your conclusion. [4]

3.3 A coin is tossed 20 times to test $H_0: p = 0.5$ against $H_1: p > 0.5$ at the 5% level, using the number of heads $X \sim B(20, 0.5)$. The critical region is $X \geq 15$.

(a) Find $P(X \geq 15)$ under H_0 and explain why $X \geq 15$ is used. [3]

(b) State what a Type I error would mean in this context. [1]

Solutions

2.1 rejecting H_0 when it is in fact true.

2.2 1.645.

3.1 B. Reject H_0 when the p -value is below the significance level 0.05.

3.2 (a) $H_0: \mu = 500$, $H_1: \mu > 500$; one-tailed.

(b) $z = \frac{502 - 500}{6/\sqrt{36}} = \frac{2}{1} = 2$; critical value 1.645; since $2 > 1.645$, reject H_0 : there is evidence the mean has increased.

3.3 (a) $P(X \geq 15) = 0.0207$ (from tables); this is the smallest "tail" region with probability ≤ 0.05 , so it is the 5% critical region.

(b) concluding the coin is biased towards heads ($p > 0.5$) when in fact it is fair.