A-LEVEL MATHEMATICS • EXERCISE SHEET

6.4 Sampling and estimation

Name: ____________ Class: ________ Date: __________ **Total: 13 marks**

KEY WORDS sample 样本 • confidence interval 置信区间 • population 总体 • central limit theorem 中心极限定理 • sample mean 样本均值

Objective

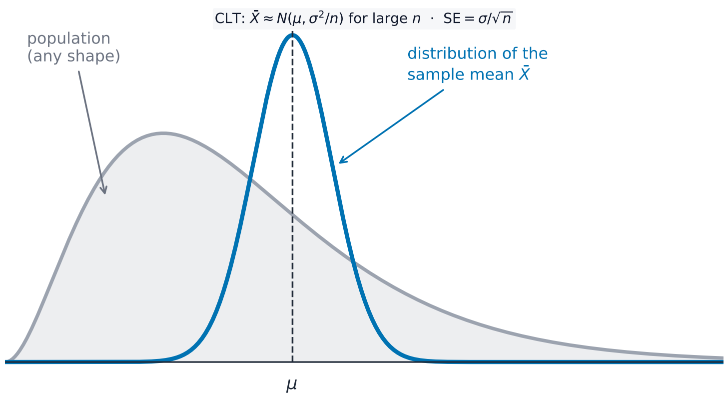
Build the skills to answer exam questions on **sampling and estimation** 抽样与估计 — the distribution of the **sample mean** 样本均值, the **Central Limit Theorem** 中心极限定理, **unbiased estimates** 无偏估计, and **confidence intervals** 置信区间.

You must be able to:

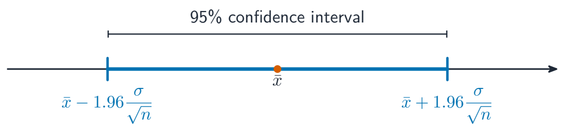
- use $E(\bar{X}) = \mu$, $\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$ and the CLT
- find unbiased estimates of the population mean and variance
- find a confidence interval $\bar{x} \pm z\dfrac{\sigma}{\sqrt{n}}$

1 Worked examples

■ Confidence interval



By the CLT, $\bar{X}$ is approximately normal for large n, centred on μ with variance σ^2/n



A 95% interval reaches 1.96 standard errors each side of $\bar{x}$

$n = 64$, $\bar{x} = 50$, $\sigma = 8$: $50 \pm 1.96\dfrac{8}{8} = 50 \pm 1.96 = (48.0, 52.0)$.

2 Practice

2.1 A population has $\sigma = 10$. For a sample of $n = 25$, find the standard error $\dfrac{\sigma}{\sqrt{n}}$. [2]

2.2 State the value of z used in a 95% confidence interval. [1]

3 Exam-style questions

3.1 As the sample size n increases, $\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$: [1]

- **A** increases
- **B** decreases
- **C** stays the same
- **D** becomes negative

3.2 A random sample of 100 light bulbs has mean lifetime $\bar{x} = 1200$ hours. The population standard deviation is $\sigma = 150$ hours.

(a) Find a 95% confidence interval for the mean lifetime. [3]

(b) State what changes to the interval if a 99% level ($z = 2.576$) is used instead. [2]

3.3 A sample of 8 values gives $\sum x = 120$ and $\sum x^2 = 1840$. Find unbiased estimates of the population mean and variance. [4]

Solutions

2.1 $\dfrac{10}{\sqrt{25}} = \dfrac{10}{5} = 2.$

2.2 1.96.

3.1 **B**. Dividing by a larger n makes $\mathrm{Var}(\bar{X})$ smaller.

3.2 (a) $1200 \pm 1.96\dfrac{150}{\sqrt{100}} = 1200 \pm 1.96(15) = 1200 \pm 29.4$; $(1170.6,\ 1229.4)$.
(b) the interval becomes **wider** (a higher confidence level needs a larger z, here 2.576).

3.3 mean $\bar{x} = \dfrac{120}{8} = 15$; unbiased variance $s^2 = \dfrac{1}{n-1}\left(\sum x^2 - \dfrac{(\sum x)^2}{n}\right) = \dfrac{1}{7}\left(1840 - \dfrac{120^2}{8}\right) = \dfrac{1}{7}(1840 - 1800) = \dfrac{40}{7} = 5.71.$