6.2 Linear combinations of random variables

Name: ____________ Class: ________ Date: __________ **Total: 14 marks**

Objective

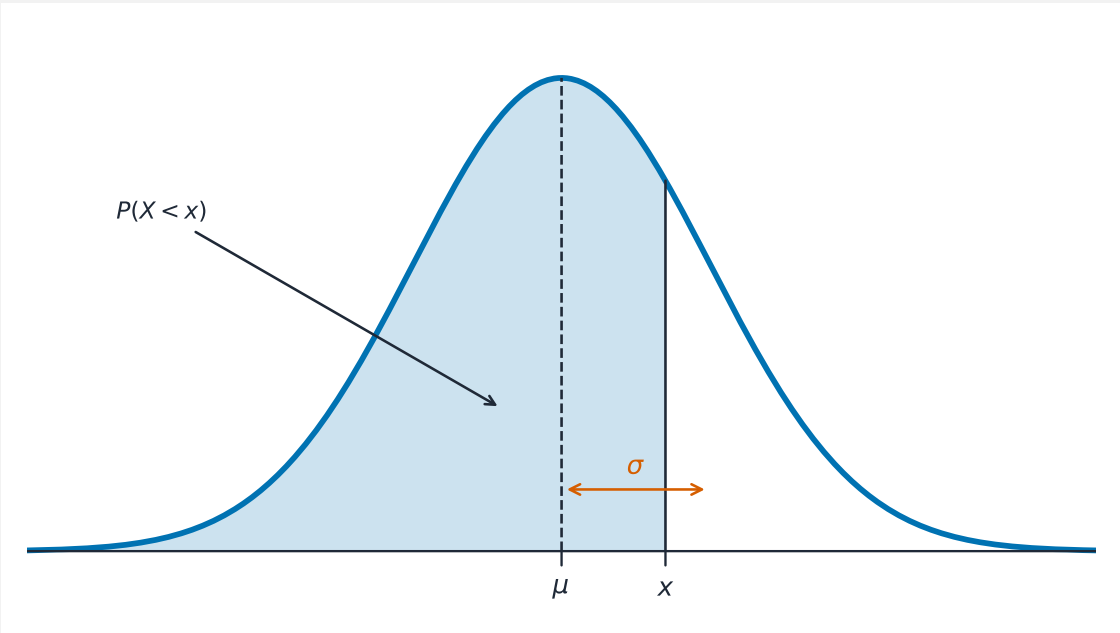
Build the skills to answer exam questions on **linear combinations of random variables 随机变量的线性组合**—the expectation and variance rules, and combining independent normal (or Poisson) variables.

You must be able to:

- use $E(aX+b)=aE(X)+b$, $\mathrm{Var}(aX+b)=a^2\mathrm{Var}(X)$
- use $E(aX+bY)=aE(X)+bE(Y)$, $\mathrm{Var}(aX+bY)=a^2\mathrm{Var}(X)+b^2\mathrm{Var}(Y)$
- recognise that a linear combination of independent normals is normal

1 Worked examples

Mean and variance rules



If X is normal then so is $aX+b$; variances add for independent variables (means always add)

X has mean 5, variance 4: $E(3X-1)=3(5)-1=14$; $\mathrm{Var}(3X-1)=3^2(4)=36$.

Watch the signs: $\mathrm{Var}(X - Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ (variances **add**).

2 Practice

2.1 X has mean 10 and variance 9. Find $E(2X + 5)$. [2]

2.2 For independent X, Y with $\mathrm{Var}(X) = 4$, $\mathrm{Var}(Y) = 9$, find $\mathrm{Var}(X + Y)$. [1]

3 Exam-style questions

3.1 X has variance 5. Then $\mathrm{Var}(4X)$ is: [1]

- **A** 20
- **B** 80
- **C** 9
- **D** 5

3.2 Independent variables have $X \sim N(20, 3^2)$ and $Y \sim N(15, 4^2)$.

(a) Find the distribution of $X - Y$. [3]

(b) Find $P(X - Y > 2)$. [3]

3.3 The mass of a tin is $X \sim N(500, 4^2)$ g. Four tins are packed in a box of mass $N(100, 2^2)$ g, all independent. Find the mean and standard deviation of the total mass of a packed box. [4]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **6.2 Linear combinations of random variables** past-paper sheet in the Library, or try these in **Practice mode**:

- 9709/61 N25 —Q1 (combining variables)
- 9709/61 N25 —Q6 (linear combination)
- 9709/62 N25 —Q2 (normal combination)

Solutions

2.1 $E(2X+5) = 2(10)+5 = 25$.

2.2 $\mathrm{Var}(X+Y) = 4+9 = 13$.

3.1 B. $\mathrm{Var}(4X) = 4^2 \times 5 = 80$.

3.2 (a) mean $= 20 - 15 = 5$; variance $= 3^2 + 4^2 = 25$; so $X - Y \sim N(5, 25)$.
(b) $Z = \dfrac{2-5}{5} = -0.6$; $P(X - Y > 2) = P(Z > -0.6) = \Phi(0.6) = 0.7257$.

3.3 total $T = X_1 + X_2 + X_3 + X_4 + B$; mean $= 4(500) + 100 = 2100$ g; variance $= 4(4^2) + 2^2 = 64 + 4 = 68$, so SD $= \sqrt{68} = 8.25$ g.