6.1 The Poisson distribution

Name: ____________ Class: ________ Date: __________ **Total: 13 marks**

Objective

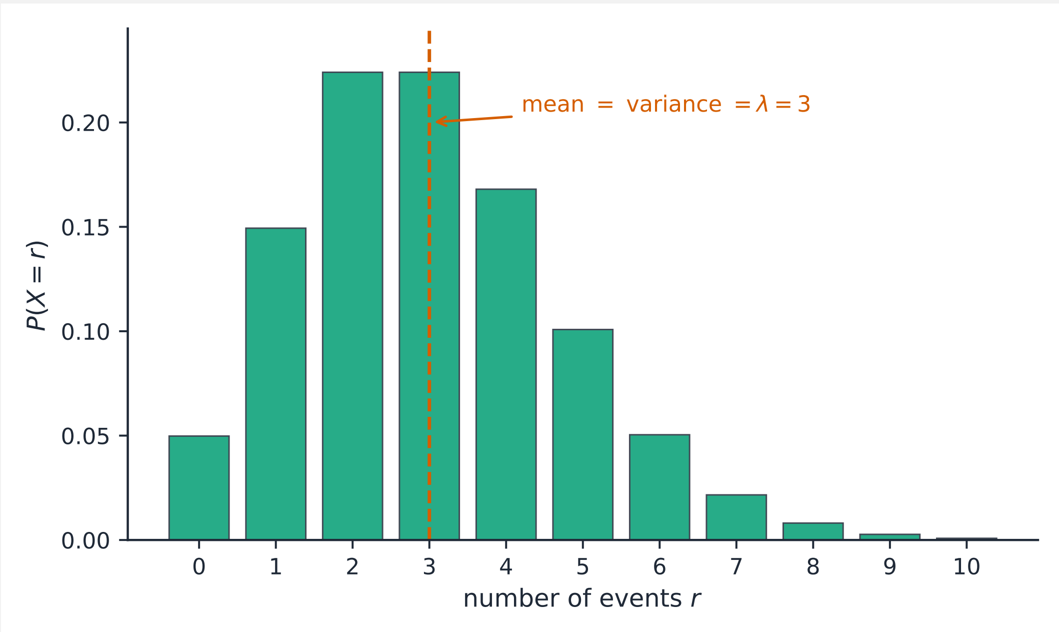
Build the skills to answer exam questions on the **Poisson distribution** 泊松分布—using Po(λ), adding Poisson variables, and the Poisson approximation to the binomial.

You must be able to:

- use $P(X = r) = e^{-\lambda}\dfrac{\lambda^r}{r!}$ (mean = variance = λ)
- combine independent Poisson variables (the sum is Poisson)
- use Po(np) to approximate $B(n, p)$ when n large, p small

1 Worked examples

■ Poisson probabilities



For a Poisson variable the mean and variance both equal λ

$X \sim \text{Po}(3)$: $P(X = 2) = e^{-3}\dfrac{3^2}{2!} = 4.5e^{-3} = 0.224$.

$P(X \geqslant 1) = 1 - P(X = 0) = 1 - e^{-3} = 0.950$.

2 Practice

2.1 $X \sim \text{Po}(2)$. Find $P(X = 0)$. [2]

2.2 State the variance of Po(4.5). [1]

3 Exam-style questions

3.1 For $X \sim \text{Po}(\lambda)$, the mean equals: [1]

- **A** λ^2
- **B** $\sqrt{\lambda}$
- **C** λ
- **D** $\dfrac{1}{\lambda}$

3.2 Calls arrive at a switchboard at a mean rate of 1.5 per minute, as a Poisson process.

(a) Find the probability of exactly 2 calls in a given minute. [2]

(b) Find the probability of at least 1 call in a 2-minute period. [3]

3.3 A factory makes items with a 0.5% defect rate. A box holds 400 items. Using a Poisson approximation, find the probability that a box contains exactly 3 defective items. [4]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **6.1 The Poisson distribution** past-paper sheet in the Library, or try these in **Practice mode**:

- 9709/61 N25 —Q1 (Poisson)
- 9709/61 N25 —Q3 (Poisson)
- 9709/62 N25 —Q1 (Poisson approximation)

Solutions

2.1 $P(X = 0) = e^{-2} = 0.135$.

2.2 variance $= \lambda = 4.5$.

3.1 **C**. For a Poisson distribution the mean is λ.

3.2 (a) $P(X = 2) = e^{-1.5}\dfrac{1.5^2}{2!} = e^{-1.5}(1.125) = 0.251$.
(b) over 2 minutes $\lambda = 3$; $P(X \geqslant 1) = 1 - e^{-3} = 1 - 0.0498 = 0.950$.

3.3 $\lambda = np = 400 \times 0.005 = 2$; $P(X = 3) = e^{-2}\dfrac{2^3}{3!} = e^{-2}\dfrac{8}{6}; = 0.1353 \times 1.333 = 0.180$.