

5.4 Discrete random variables

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

KEY WORDS geometric 几何分布 • discrete random variable 离散型随机变量
• binomial 二项分布 • variance 方差 • expectation 期望

Objective

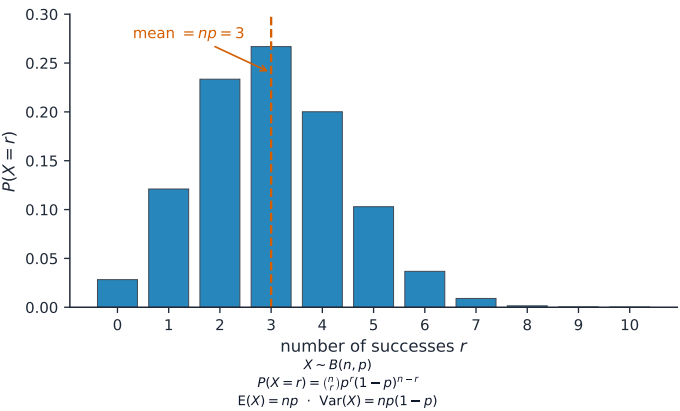
Build the skills to answer exam questions on **discrete random variables** 离散型随机变量 — probability distributions, **expectation** 期望 and **variance** 方差, the **binomial** 二项分布 and **geometric** 几何分布 distributions.

You must be able to:

- use a distribution table (probabilities sum to 1); find $E(X)$ and $\text{Var}(X)$
- use $B(n, p)$: $P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$, $E(X) = np$
- use the geometric model $P(X = r) = (1 - p)^{r-1} p$, $E(X) = \frac{1}{p}$

1 Worked examples

■ Binomial distribution



$B(10, 0.3)$: each bar is $P(X = r)$, clustered around the mean $np = 3$

$X \sim B(10, 0.3)$: $P(X = 2) = \binom{10}{2} (0.3)^2 (0.7)^8 = 0.233$; $E(X) = np = 3$.

Distribution: $E(X) = \sum xP(X=x)$, $\text{Var}(X) = \sum x^2P(X=x) - [E(X)]^2$.

2 Practice

2.1 $X \sim B(8, 0.25)$. Find $E(X)$. [1]

2.2 A geometric variable has $p = 0.2$. Find $E(X)$. [1]

3 Exam-style questions

3.1 For $X \sim B(12, 0.5)$, the variance $\text{Var}(X)$ is: [1]

- A 6
- B 3
- C 12
- D 1.5

3.2 A discrete random variable X has the distribution:

x	1	2	3	4
$P(X = x)$	0.1	0.3	k	0.2

(a) Find k . [1]

(b) Find $E(X)$ and $\text{Var}(X)$. [5]

3.3 A fair coin is tossed 6 times. Find the probability of obtaining exactly 4 heads. [3]

Solutions

2.1 $E(X) = np = 8 \times 0.25 = 2.$

2.2 $E(X) = \frac{1}{p} = \frac{1}{0.2} = 5.$

3.1 B. $\text{Var}(X) = np(1 - p) = 12(0.5)(0.5) = 3.$

3.2 (a) probabilities sum to 1: $0.1 + 0.3 + k + 0.2 = 1 \Rightarrow k = 0.4.$

(b) $E(X) = 1(0.1) + 2(0.3) + 3(0.4) + 4(0.2) = 0.1 + 0.6 + 1.2 + 0.8 = 2.7$; $E(X^2) = 1(0.1) + 4(0.3) + 9(0.4) + 16(0.2) = 0.1 + 1.2 + 3.6 + 3.2 = 8.1$; $\text{Var}(X) = 8.1 - 2.7^2 = 8.1 - 7.29 = 0.81.$

3.3 $X \sim B(6, 0.5)$; $P(X = 4) = \binom{6}{4}(0.5)^6 = 15 \times \frac{1}{64} = \frac{15}{64} = 0.234.$