A-LEVEL MATHEMATICS • EXERCISE SHEET

5.4 Discrete random variables

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

Objective

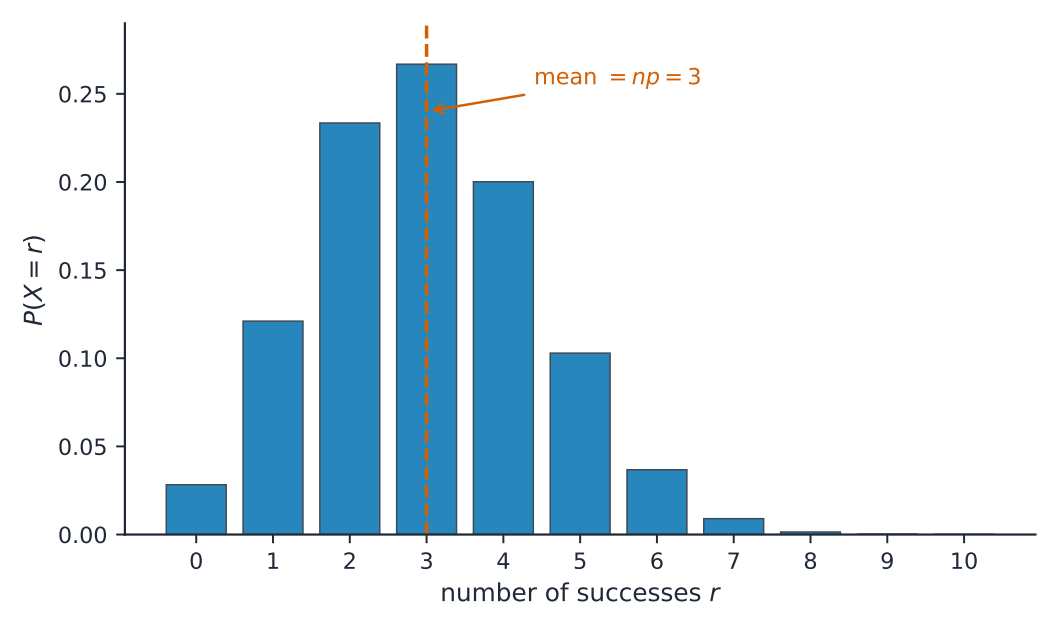
Build the skills to answer exam questions on **discrete random variables** 离散型随机变量—probability distributions, **expectation** 期望 and **variance** 方差, the **binomial** 二项分布 and **geometric** 几何分布 distributions.

You must be able to:

- use a distribution table (probabilities sum to 1); find $E(X)$ and $\mathrm{Var}(X)$
- use $B(n,p)$: $P(X=r)=\binom{n}{r}p^r(1-p)^{n-r}$, $E(X)=np$
- use the geometric model $P(X=r)=(1-p)^{r-1}p$, $E(X)=\frac{1}{p}$

1 Worked examples

Binomial distribution



$B(10,0.3)$: each bar is $P(X=r)$, clustered around the mean $np=3$

$X\sim B(10,0.3)$**:** $P(X=2)=\binom{10}{2}(0.3)^2(0.7)^8=0.233$; $E(X)=np=3$.

Distribution: $E(X)=\sum xP(X{=}x)$, $\mathrm{Var}(X)=\sum x^2P(X{=}x)-[E(X)]^2$.

2 Practice

2.1 $X \sim B(8, 0.25)$. Find $E(X)$. [1]

2.2 A geometric variable has $p = 0.2$. Find $E(X)$. [1]

3 Exam-style questions

3.1 For $X \sim B(12, 0.5)$, the variance $\text{Var}(X)$ is: [1]

- **A** 6
- **B** 3
- **C** 12
- **D** 1.5

3.2 A discrete random variable X has the distribution:

x	1	2	3	4
$P(X = x)$	0.1	0.3	k	0.2

(a) Find k. [1]

(b) Find $E(X)$ and $\text{Var}(X)$. [5]

3.3 A fair coin is tossed 6 times. Find the probability of obtaining exactly 4 heads. [3]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **5.4 Discrete random variables** past-paper sheet in the Library, or try these in **Practice mode**:

- 9709/51 N25 —Q1 (binomial)
- 9709/51 N25 —Q5 (distribution / expectation)
- 9709/52 N25 —Q1 (discrete random variable)

Solutions

2.1 $E(X) = np = 8 \times 0.25 = 2$.

2.2 $E(X) = \dfrac{1}{p} = \dfrac{1}{0.2} = 5$.

3.1 B. $\text{Var}(X) = np(1-p) = 12(0.5)(0.5) = 3$.

3.2 (a) probabilities sum to 1: $0.1 + 0.3 + k + 0.2 = 1 \Rightarrow k = 0.4$.
(b) $E(X) = 1(0.1) + 2(0.3) + 3(0.4) + 4(0.2) = 0.1 + 0.6 + 1.2 + 0.8 = 2.7$; $E(X^2) = 1(0.1) + 4(0.3) + 9(0.4) + 16(0.2) = 0.1 + 1.2 + 3.6 + 3.2 = 8.1$; $\text{Var}(X) = 8.1 - 2.7^2 = 8.1 - 7.29 = 0.81$.

3.3 $X \sim B(6, 0.5)$; $P(X = 4) = \binom{6}{4}(0.5)^6 = 15 \times \dfrac{1}{64} = \dfrac{15}{64} = 0.234$.