

5.2 Permutations and combinations

Name: _____ Class: _____ Date: _____

Total: 11 marks

Objective

Build the skills to answer exam questions on **permutations and combinations** 排列与组合—counting arrangements (order matters) and selections (order does not), including repeated objects and restrictions.

You must be able to:

- use ${}^n P_r$ for arrangements and ${}^n C_r$ for selections
- arrange objects with repeats (divide by each repeat factorial)
- count with restrictions (e.g. items together / not together)

1 Worked examples

■ Arrangements & selections



Counting outcomes underlies both combinatorics and probability

Image: source

Arranging 3 of 4 objects (4P_3)

$$\boxed{4} \times \boxed{3} \times \boxed{2} = 24$$

1st pick 2nd pick 3rd pick

Choosing 3 of 4 (order ignored): ${}^4C_3 = \frac{{}^4P_3}{3!} = 4$

Arrange (order matters, nP_r) versus choose (order ignored, nC_r)

Arrangements of NEEDLESS (8 letters, E $\times$ 3, S $\times$ 2): $\frac{8!}{3!2!} = 3360$.

Choosing 3 from 10 (order doesn't matter): ${}^{10}C_3 = \frac{10!}{3!7!} = 120$.

2 Practice

2.1 Evaluate 7P_3 . [1]

2.2 Evaluate 8C_2 . [1]

3 Exam-style questions

3.1 The number of ways to arrange the letters of BANANA is: [1]

- A 720
- B 120
- C 60
- D 360

3.2 A committee of 4 is chosen from 7 women and 5 men.

(a) Find the number of possible committees. [2]

(b) Find the number that contain exactly 2 women and 2 men. [3]

3.3 Find the number of arrangements of the 7 letters of PICTURE in which the three vowels (I, U, E) are all together. [3]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **5.2 Permutations and combinations** past-paper sheet in the Library, or try these in **Practice mode**:

- 9709/51 N25 —Q7 (permutations / combinations)
- 9709/52 N25 —Q7 (counting)
- 9709/51 J25 —Q2 (arrangements)

Solutions

2.1 ${}^7P_3 = 7 \times 6 \times 5 = 210.$

2.2 ${}^8C_2 = \frac{8 \times 7}{2} = 28.$

3.1 C. BANANA has 6 letters with A $\times$ 3, N $\times$ 2: $\frac{6!}{3!2!} = \frac{720}{12} = 60.$

3.2 (a) ${}^{12}C_4 = \frac{12 \cdot 11 \cdot 10 \cdot 9}{24} = 495.$

(b) ${}^7C_2 \times {}^5C_2 = 21 \times 10 = 210.$

3.3 treat (IUE) as one block $\rightarrow$ 5 items arrange in $5! = 120$ ways; the vowels arrange among themselves in $3! = 6$ ways; total = $120 \times 6 = 720.$