A-LEVEL MATHEMATICS • EXERCISE SHEET

4.1 Forces and equilibrium

Name: ____________ Class: ________ Date: __________ **Total: 32 marks**

KEY WORDS friction 摩擦 • equilibrium 平衡 • coefficient of friction 摩擦系数 • limiting equilibrium 极限平衡 • forces and equilibrium 力与平衡

Objective

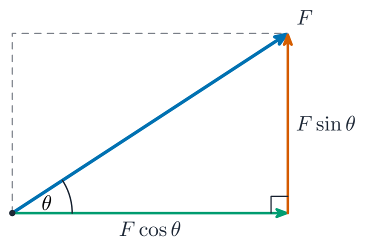
Build the skills to answer exam questions on **forces and equilibrium 力与平衡** — resolving forces into **components 分量**, the condition for **equilibrium 平衡**, and **friction 摩擦** ($F \leqslant \mu R$). Take $g = 10$ m s^{-2}.

You must be able to:

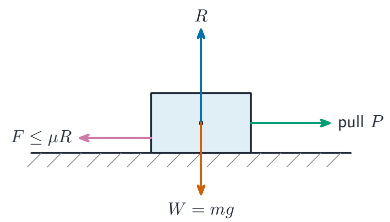
- resolve a force into horizontal and vertical components
- use "components in any direction sum to zero" for equilibrium
- use $F \leqslant \mu R$ and limiting friction $F = \mu R$

1 Worked examples

Resolving & friction



A force at angle θ has parts $F\cos\theta$ (horizontal) and $F\sin\theta$ (vertical)



The contact force splits into the normal reaction R and friction $F \leqslant \mu R$

A 5 kg block rests on a rough horizontal floor ($\mu = 0.3$). Max friction $= \mu R = 0.3 \times 50 = 15$ N — a horizontal pull below 15 N will not move it.

Resolving, and the two conditions for equilibrium

A force at an angle is replaced by two perpendicular **components**: along a chosen direction it is $F\cos\theta$, perpendicular to it $F\sin\theta$, where θ is measured **from that direction**. Getting the cosine and sine the right way round is the single commonest error; check by asking whether the component should vanish when $\theta = 90°$.

Equilibrium means the resultant is zero, which gives two equations:

$$\sum F_x = 0 \qquad \sum F_y = 0$$

Choose the axes to kill a force. On an inclined plane, resolving **along** and **perpendicular to** the slope removes the need to resolve the normal contact force and the friction at all — only the weight has to be split, into $mg\sin\alpha$ down the slope and $mg\cos\alpha$ into it.

Friction. $F \leqslant \mu R$, with **equality only when the object is on the point of sliding** ("limiting equilibrium"). Writing $F = \mu R$ for an object that is merely stationary is wrong, and it is a mark.

Worked example. A box of mass 8 kg rests on a rough plane inclined at 20°, held by a rope along the slope. The coefficient of friction is 0.3 and the box is on the point of sliding **down**. Find the tension.

1. Take axes along and perpendicular to the slope; $g = 9.81$.
2. Perpendicular: $R = mg\cos 20° = 8(9.81)\cos 20° = 73.8$ N.
3. On the point of sliding down, so friction acts **up** the slope at its maximum: $F = \mu R = 0.3(73.8) = 22.1$ N.
4. Along the slope: $T + F = mg\sin 20°$, so $T = 8(9.81)\sin 20° - 22.1 = 26.8 - 22.1 = 4.7$ N.

△ **Friction opposes the motion that would happen**, so its direction depends on which way the object is about to slide. If the box were about to slide **up**, friction would act down the slope and the tension would be $26.8 + 22.1 = 48.9$ N — the same numbers, a completely different answer.

2 Practice

2.1 A force of 20 N acts at 60° above the horizontal. Find its horizontal component. [2]

2.2 A block of weight 40 N rests on a rough surface, $\mu = 0.25$. Find the maximum friction available. [2]

2.3 A force of 20 N acts at 35° above the horizontal. Find its horizontal and vertical components. [2]

2.4 A block of mass 5 kg rests on a rough horizontal table with $\mu = 0.4$. Find the least horizontal force that would move it. [3]

2.5 State the condition under which $F = \mu R$ rather than $F \leqslant \mu R$. [1]

3 Exam-style questions

3.1 A particle is in equilibrium under three forces. The sum of the forces is: [1]

- **A** maximum
- **B** zero
- **C** equal to the weight
- **D** equal to μR

3.2 A box of mass 8 kg rests on a rough horizontal floor. A horizontal force of 30 N is applied but the box does not move.

(a) Find the normal reaction. [1]

(b) Given the box is on the point of slipping, find the coefficient of friction. [3]

3.3 A particle of weight W hangs in equilibrium from two light strings making angles 30° and 60° with the horizontal. The tensions are T_1 and T_2. Given $W = 20$ N, find T_1 and T_2. [5]

3.4 A particle of mass 6 kg rests on a rough plane inclined at 25° to the horizontal. The coefficient of friction between the particle and the plane is 0.35. A force P acts on the particle **up the line of greatest slope**. Take $g = 9.81$ m s^{-2}.

(a) Find the normal contact force. [2]

(b) Find the value of P for which the particle is on the point of sliding **down** the plane. [4]

(c) Find the value of P for which the particle is on the point of sliding **up** the plane. [3]

(d) Hence state the range of values of P for which the particle remains in equilibrium. [1]

(e) Explain why the friction force changes direction between parts (b) and (c). [2]

Solutions

2.1 $20\cos 60° = 20 \times 0.5 = 10$ N.

2.2 $F_{\max} = \mu R = 0.25 \times 40 = 10$ N.

2.3 Horizontal $= 20\cos 35° = 16.4$ N; vertical $= 20\sin 35° = 11.5$ N.

2.4 $R = 5(9.81) = 49.05$ N; $F = \mu R = 0.4(49.05) = 19.6$ N.

2.5 When the object is in limiting equilibrium — on the point of sliding.

3.1 B. In equilibrium the resultant (vector sum) of the forces is zero.

3.2 (a) $R = mg = 8 \times 10 = 80$ N.
(b) on the point of slipping $F = \mu R$ and $F = 30$ N; $\mu = \dfrac{30}{80}$; $= 0.375$.

3.3 resolve: horizontal $T_2\cos 60° = T_1\cos 30°$; vertical $T_1\sin 30° + T_2\sin 60° = 20$; from the first $T_2(0.5) = T_1(0.866) \Rightarrow T_2 = 1.732\,T_1$; substitute: $0.5T_1 + 0.866(1.732T_1) = 20 \Rightarrow 2T_1 = 20 \Rightarrow T_1 = 10$ N, $T_2 = 17.3$ N.

3.4 (a) Resolving perpendicular to the plane: $R = 6(9.81)\cos 25° = 53.3$ N.
(b) On the point of sliding down, friction acts up the plane at its maximum: $F = \mu R = 0.35(53.3) = 18.7$ N; resolving along the plane, $P + F = mg\sin 25°$; $mg\sin 25° = 6(9.81)\sin 25° = 24.9$ N; $P = 24.9 - 18.7 = 6.2$ N.
(c) On the point of sliding up, friction acts down the plane: $P = mg\sin 25° + F = 24.9 + 18.7 = 43.6$ N.
(d) $6.2 \text{ N} \leqslant P \leqslant 43.6$ N.
(e) Friction always opposes the motion that would occur if it were absent; with a small P the particle would slide down so friction acts up the plane, and with a large P it would slide up so friction acts down.