

3.9 Complex numbers

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS complex numbers 复数 • argand diagram 阿干图 • argument 辐角 • loci 轨迹
• modulus 模

Objective

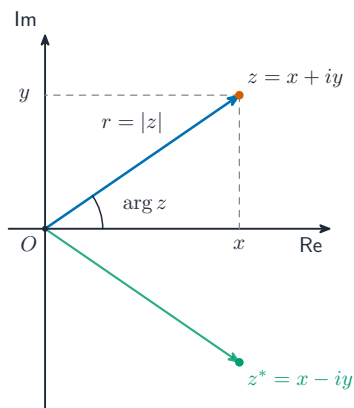
Build the skills to answer exam questions on **complex numbers** 复数 — arithmetic and **conjugates** 共轭, **modulus** 模 and **argument** 辐角, the **Argand diagram** 阿干图, and **loci** 轨迹.

You must be able to:

- add, multiply and divide complex numbers (divide using the conjugate)
- find modulus and argument; convert to polar form
- sketch loci such as $|z - a| = r$, $|z - a| = |z - b|$, $\arg(z - a) = \theta$

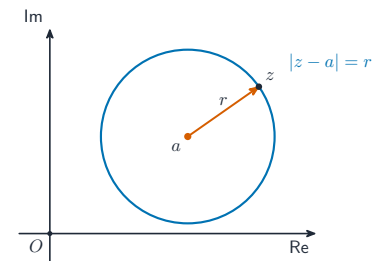
1 Worked examples

■ Division & loci



$|z|$ is the distance from O , $\arg z$ the angle, z^* the reflection in the real axis

$$\frac{3+i}{1-i} = \frac{(3+i)(1+i)}{2} = \frac{2+4i}{2} = 1+2i.$$



$|z - a| = r$ is a circle of radius r centred at a

2 Practice

2.1 Find the modulus of $z = 3 + 4i$. [1]

2.2 Write $(2 + i)(3 - i)$ in the form $x + iy$. [2]

3 Exam-style questions

3.1 The argument of $z = -1 + i$ (in radians, $-\pi < \arg z \leq \pi$) is: [1]

- A $\frac{\pi}{4}$
- B $\frac{3\pi}{4}$
- C $-\frac{\pi}{4}$
- D $\frac{\pi}{2}$

3.2 The complex number z satisfies $z = \frac{10}{2-i}$.

(a) Express z in the form $x + iy$. [3]

(b) Find $|z|$ and $\arg z$ (in radians). [3]

3.3 On a single Argand diagram, sketch the locus of points z for which:

(a) $|z - (2 + i)| = 2$. [2]

(b) Describe the locus $|z - 3| = |z + 1|$. [2]

Solutions

2.1 $|z| = \sqrt{3^2 + 4^2} = 5$.

2.2 $(2 + i)(3 - i) = 6 - 2i + 3i - i^2 = 7 + i$.

3.1 B. The point $(-1, 1)$ is in the second quadrant: $\arg z = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

3.2 (a) $z = \frac{10(2+i)}{(2-i)(2+i)} = \frac{20+10i}{5} = 4 + 2i$.

(b) $|z| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$; $\arg z = \tan^{-1} \frac{2}{4} = 0.4636$ rad.

3.3 (a) a circle of radius 2 centred at the point $(2, 1)$.

(b) the perpendicular bisector of the segment joining 3 and -1 — i.e. the vertical line $x = 1$.