

3.8 Differential equations

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS general solution 通解 • initial condition 初始条件 • particular solution 特解
• differential equations 微分方程 • separable 可分离变量

Objective

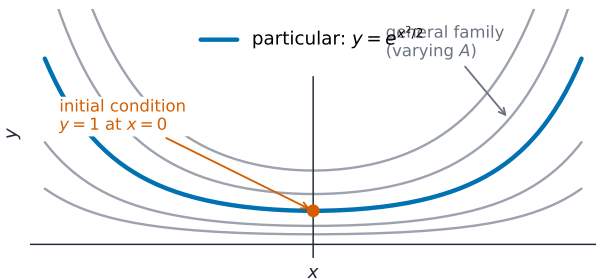
Build the skills to answer exam questions on **differential equations** 微分方程 — solving a **separable** 可分离变量 first-order equation, finding the **general** 通解 and **particular solution** 特解, and setting up a model.

You must be able to:

- separate variables and integrate both sides
- use an initial condition to find the constant
- form and solve a differential equation from a rate description

1 Worked examples

■ Separate and solve



general: family with A • IC $\rightarrow$ particular solution

The constant gives a family of curves; an initial condition picks one particular solution

Solve $\frac{dy}{dx} = xy, y = 1$ at $x = 0$.

$\int \frac{1}{y} dy = \int x dx \Rightarrow \ln y = \frac{1}{2}x^2 + c \Rightarrow y = Ae^{x^2/2}; \quad y(0) = 1 \Rightarrow A = 1, \text{ so } y = e^{x^2/2}.$

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2 Practice

2.1 Separate the variables for $\frac{dy}{dx} = \frac{y}{x}$ (write each side ready to integrate). [1]

2.2 Solve $\frac{dy}{dx} = 3y$, giving the general solution. [2]

3 Exam-style questions

3.1 The general solution of $\frac{dy}{dx} = ky$ is: [1]

- A $y = kx + c$
- B $y = Ae^{kx}$
- C $y = Ax^k$
- D $y = \frac{1}{2}kx^2 + c$

3.2 Solve $\frac{dy}{dx} = 2x(y + 1)$, given $y = 0$ when $x = 0$. Give y explicitly. [5]

3.3 A tank loses water so that the depth h falls at a rate proportional to $\sqrt{h}$: $\frac{dh}{dt} = -k\sqrt{h}$. Initially $h = H$.

(a) Solve to show $\sqrt{h} = \sqrt{H} - \frac{1}{2}kt$. [4]

(b) Find, in terms of H and k , the time for the tank to empty. [2]

Solutions

2.1 $\frac{1}{y} dy = \frac{1}{x} dx$.

2.2 $\ln y = 3x + c \Rightarrow y = Ae^{3x}$.

3.1 B. $\frac{dy}{dx} = ky$ gives $y = Ae^{kx}$.

3.2 $\int \frac{1}{y+1} dy = \int 2x dx$; $\ln(y+1) = x^2 + c$; $y(0) = 0 \Rightarrow c = 0$; $y+1 = e^{x^2}$; $y = e^{x^2} - 1$.

3.3 (a) $\int h^{-1/2} dh = \int -k dt$; $2\sqrt{h} = -kt + c$; $h = H$ at $t = 0 \Rightarrow c = 2\sqrt{H}$; $\sqrt{h} = \sqrt{H} - \frac{1}{2}kt$.

(b) empty when $h = 0$: $0 = \sqrt{H} - \frac{1}{2}kt \Rightarrow t = \frac{2\sqrt{H}}{k}$.