A-LEVEL MATHEMATICS • EXERCISE SHEET

3.7 Vectors

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS vectors 向量 • position vector 位置向量 • scalar product 数量积 • scalar 标量 • magnitude 模长

Objective

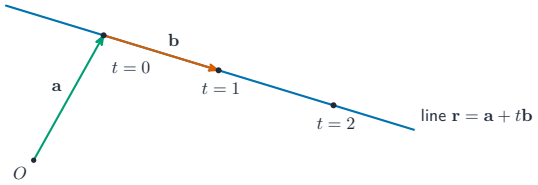
Build the skills to answer exam questions on **vectors** 向量 — magnitude and unit vectors, the **vector equation of a line** 直线向量方程, the **scalar product** 数量积, and the angle between lines.

You must be able to:

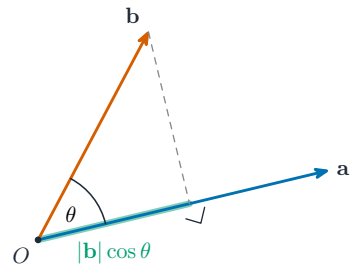
- find magnitude, a unit vector and a displacement $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$
- write and use $\mathbf{r} = \mathbf{a} + t\mathbf{b}$; test for intersection
- use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$ to find angles / perpendicularity

1 Worked examples

■ **Line and scalar product**



$\mathbf{r} = \mathbf{a} + t\mathbf{b}$*: start at* $\mathbf{a}$*, add* t *copies of the direction* $\mathbf{b}$



$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$*; it is* 0 *when the vectors are perpendicular*

Angle between $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$: $\mathbf{a} \cdot \mathbf{b} = 8$, $|\mathbf{a}| = |\mathbf{b}| = 3$, so $\cos\theta = \frac{8}{9}$, $\theta = 27.3°$.

2 Practice

2.1 Find $|\mathbf{v}|$ for $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$. [2]

2.2 Find $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$. [2]

3 Exam-style questions

3.1 Two vectors are perpendicular when their scalar product is: [1]

- **A** 1
- **B** 0
- **C** the product of their magnitudes
- **D** undefined

3.2 The line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

(a) Find the position vector of the point on l where $t = -3$. [2]

(b) Find the acute angle between l and the direction $\mathbf{d} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$. [4]

3.3 Points $A(1, 2, 1)$, $B(3, 3, 4)$ and $C(2, 0, 3)$ are given.

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2]

(b) Find angle BAC. [3]

Solutions

2.1 $|\mathbf{v}| = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$.

2.2 $\mathbf{a} \cdot \mathbf{b} = 2 - 3 + 1 = 0$.

3.1 **B**. Perpendicular vectors have scalar product 0.

3.2 (a) $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} - 3 \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -7 \\ 9 \end{pmatrix}$.
(b) direction of l is $\mathbf{b} = (1, 2, -2)$; $\mathbf{b} \cdot \mathbf{d} = (1)(2) + (2)(-1) + (-2)(2) = 2 - 2 - 4 = -4$; $|\mathbf{b}| = \sqrt{1 + 4 + 4} = 3$ and $|\mathbf{d}| = \sqrt{4 + 1 + 4} = 3$; $\cos\theta = \dfrac{-4}{9}$, so $\theta = 116.4°$ — obtuse. The **acute** angle between the two directions is therefore $180° - 116.4° = 63.6°$; equivalently, take the modulus first: $\cos\theta = \frac{4}{9}$.

3.3 (a) $\overrightarrow{AB} = (2, 1, 3)$, $\overrightarrow{AC} = (1, -2, 2)$.
(b) $\overrightarrow{AB} \cdot \overrightarrow{AC} = 2 - 2 + 6 = 6$; $|\overrightarrow{AB}| = \sqrt{14}$, $|\overrightarrow{AC}| = 3$; $\cos(BAC) = \dfrac{6}{3\sqrt{14}} = 0.5345 \Rightarrow BAC = 57.7°$.