A-LEVEL MATHEMATICS • EXERCISE SHEET

3.4 Differentiation (inverse tangent, implicit, parametric)

Name: ____________ Class: ________ Date: __________ **Total: 14 marks**

KEY WORDS quotient 商 • inverse tangent 反正切 • differentiation 微分

Objective

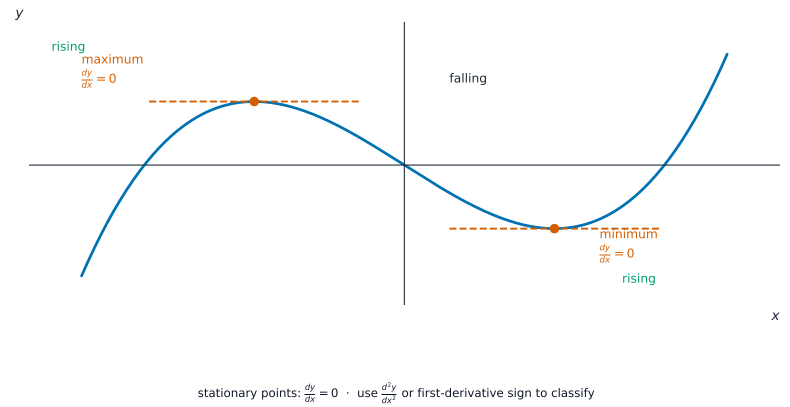
Build the skills to answer exam questions on **differentiation** 微分 in Pure 3 — the Pure 2 rules plus the **inverse tangent** 反正切 derivative, applied to parametric and implicit curves.

You must be able to:

- use $\dfrac{d}{dx}\tan^{-1}x = \dfrac{1}{1+x^2}$
- apply the product, quotient and chain rules
- differentiate parametric and implicit relations

1 Worked examples

■ Mixed rules



Stationary points and gradients still come from differentiating — now with $\tan^{-1}$ *and implicit methods too*

$y = \tan^{-1}(2x)$**:** chain rule $\to \dfrac{dy}{dx} = \dfrac{1}{1+(2x)^2}\cdot 2 = \dfrac{2}{1+4x^2}$.

Implicit $e^{2y} = x + y$**:** $2e^{2y}\dfrac{dy}{dx} = 1 + \dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2e^{2y}-1}$.

2 Practice

2.1 Differentiate $y = \tan^{-1}(x^2)$. [2]

2.2 Differentiate $y = x\ln x$. [2]

3 Exam-style questions

3.1 $\dfrac{d}{dx}\tan^{-1}(3x)$ equals: [1]

- **A** $\dfrac{3}{1+9x^2}$
- **B** $\dfrac{1}{1+3x^2}$
- **C** $\dfrac{3}{1+3x^2}$
- **D** $\dfrac{1}{1+9x^2}$

3.2 A curve has parametric equations $x = 1 + 2\cos t$, $y = 3\sin t$ for $0 \leqslant t \leqslant \pi$. Find $\dfrac{dy}{dx}$ in terms of t and the gradient at $t = \frac{\pi}{6}$. [4]

3.3 A curve is given implicitly by $x^2y + y^3 = 10$.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y. [3]

(b) Find the gradient at the point $(1, 2)$. [2]

Solutions

2.1 $\dfrac{dy}{dx} = \dfrac{2x}{1+x^4}$.

2.2 $\dfrac{dy}{dx} = \ln x + 1$.

3.1 A. $\dfrac{1}{1+(3x)^2}\cdot 3 = \dfrac{3}{1+9x^2}$.

3.2 $\dfrac{dx}{dt} = -2\sin t$, $\dfrac{dy}{dt} = 3\cos t$; $\dfrac{dy}{dx} = \dfrac{3\cos t}{-2\sin t} = -\frac{3}{2}\cot t$; at $t = \frac{\pi}{6}$: $-\frac{3}{2}\cot 30^\circ = -\frac{3}{2}\sqrt{3} = -\frac{3\sqrt{3}}{2}$.

3.3 (a) $2xy + x^2\dfrac{dy}{dx} + 3y^2\dfrac{dy}{dx} = 0$; $\dfrac{dy}{dx} = -\dfrac{2xy}{x^2+3y^2}$.

(b) at $(1, 2)$: $-\dfrac{2(1)(2)}{1+12} = -\dfrac{4}{13}$.