A-LEVEL MATHEMATICS • EXERCISE SHEET

3.3 Trigonometry (identities, R-formula)

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS r-formula 辅助角公式 • trigonometry 三角学

Objective

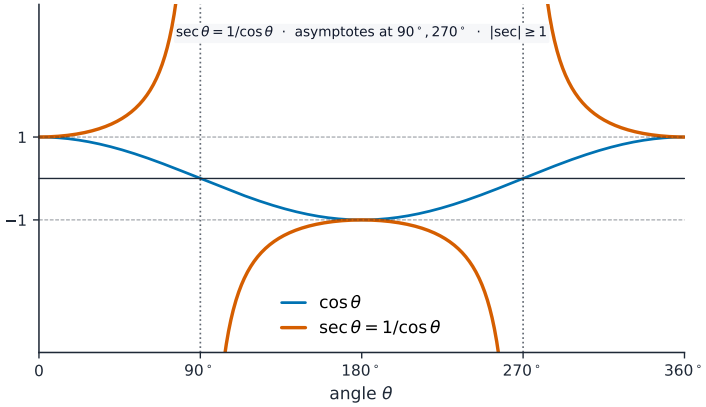
Build the skills to answer exam questions on **trigonometry** 三角学 in Pure 3 — the reciprocal identities, compound- and double-angle formulae, and the **R-formula** 辅助角公式, used in proofs and equations.

You must be able to:

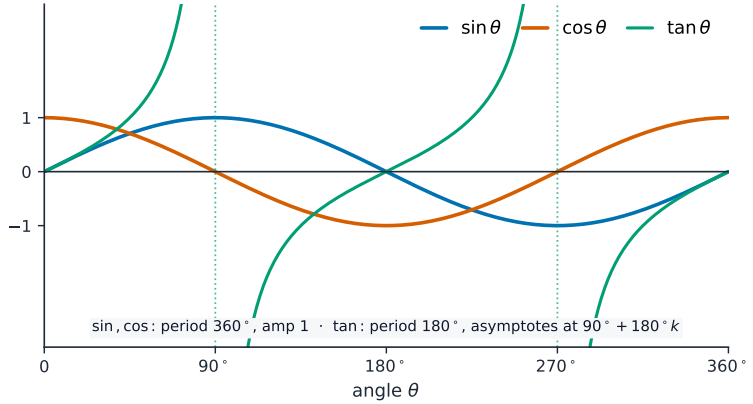
- prove identities using $\sec^2\theta \equiv 1 + \tan^2\theta$ etc.
- expand and use compound/double-angle formulae
- express $a\sin\theta + b\cos\theta$ as $R\sin(\theta \pm \alpha)$ and solve

1 Worked examples

■ R-formula in an equation



The reciprocal functions and the identities let you reduce an equation to one function



The R-formula rewrites a sum as a single sinusoid

Solve $3\sin\theta + 4\cos\theta = 2$ **for** $0° \leqslant \theta \leqslant 360°$. Write LHS as $5\sin(\theta + 53.13°)$: $\sin(\theta + 53.13°) = 0.4$, so $\theta + 53.13° = 23.6°$ or $156.4°$ $(+360°)$. Taking those in range gives $\theta = 103.3°$ or $330.5°$.

2 Practice

2.1 Write $\cos 2\theta$ in terms of $\cos\theta$ only. [1]

2.2 Find R for $7\sin\theta + 24\cos\theta = R\sin(\theta + \alpha)$. [1]

3 Exam-style questions

3.1 Using $\sin 2\theta = 2\sin\theta\cos\theta$, the value of $\sin 2\theta$ when $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$ is: [1]

- **A** $\frac{12}{25}$
- **B** $\frac{24}{25}$
- **C** $\frac{7}{25}$
- **D** $\frac{6}{5}$

3.2 (a) Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, with $R > 0$ and $0° < \alpha < 90°$. [3]

(b) Hence solve $4\sin\theta - 3\cos\theta = 2$ for $0° \leqslant \theta \leqslant 360°$. [4]

3.3 Prove the identity $\dfrac{\sin 2\theta}{1 + \cos 2\theta} \equiv \tan\theta$. [3]

Solutions

2.1 $\cos 2\theta = 2\cos^2\theta - 1$.

2.2 $R = \sqrt{7^2 + 24^2} = 25$.

3.1 B. $2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$.

3.2 (a) $R = \sqrt{4^2 + 3^2} = 5$; $\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.9°$; $5\sin(\theta - 36.9°)$.
(b) $\sin(\theta - 36.9°) = 0.4 \Rightarrow \theta - 36.9° = 23.6°$ or $156.4°$; $\theta = 60.5°$ or $193.3°$.

3.3 $\sin 2\theta = 2\sin\theta\cos\theta$ and $1 + \cos 2\theta = 2\cos^2\theta$; ratio $= \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$.