A-LEVEL MATHEMATICS • EXERCISE SHEET

3.2 Logarithmic and exponential functions

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS logarithmic and exponential functions 对数与指数函数

Objective

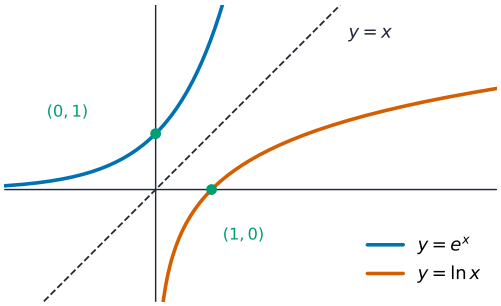
Build the skills to answer exam questions on **logarithmic and exponential functions** 对数与指数函数 in Pure 3 — the same log/exp toolkit as Pure 2, applied to equations and modelling.

You must be able to:

- use the laws of logarithms and e^x/$\ln x$ as inverses
- solve equations with the unknown in the power or inside a log
- handle exponential models (growth/decay)

1 Worked examples

■ Solving log/exp equations



ln and exp inverse · reflect in $y = x$

e^x and $\ln x$ undo each other — take logs to free an unknown power

Solve $e^{2x} - 5e^x + 6 = 0$**.** Let $u = e^x$: $u^2 - 5u + 6 = 0 \Rightarrow (u-2)(u-3) = 0$. So $e^x = 2$ or 3, giving $x = \ln 2$ or $\ln 3$.

2 Practice

2.1 Solve $\ln(x-1) = 2$, giving x to 3 s.f. [2]

2.2 Write $2\ln a - \ln b$ as a single logarithm. [1]

3 Exam-style questions

3.1 The solution of $5e^{-x} = 2$ is: [1]

- **A** $\ln 2.5$
- **B** $-\ln 2.5$
- **C** $\ln 0.4$
- **D** $\ln 10$

3.2 Solve the equation $e^{2x} - 7e^x + 10 = 0$, giving exact answers. [4]

3.3 A quantity decays so that $P = P_0 e^{-kt}$. After 5 years P has fallen to half of P_0.

(a) Find k, to 3 s.f. [3]

(b) Find the time for P to fall to one tenth of P_0. [2]

Solutions

2.1 $x - 1 = e^2 \Rightarrow x = 1 + e^2 = 8.39$.

2.2 $\ln \dfrac{a^2}{b}$.

3.1 A. $e^{-x} = 0.4 \Rightarrow -x = \ln 0.4 \Rightarrow x = -\ln 0.4 = \ln 2.5$.

3.2 let $u = e^x$: $u^2 - 7u + 10 = 0 \Rightarrow (u-2)(u-5) = 0$; $e^x = 2$ or 5; $x = \ln 2$ or $\ln 5$.

3.3 (a) $\frac{1}{2}P_0 = P_0 e^{-5k} \Rightarrow e^{-5k} = 0.5 \Rightarrow -5k = \ln 0.5$; $k = 0.139$.

(b) $0.1 = e^{-kt} \Rightarrow t = \dfrac{\ln 0.1}{-0.139} = 16.6$ years.