A-LEVEL MATHEMATICS • EXERCISE SHEET

# 2.3 Trigonometry (reciprocal, compound, double angle, R-formula)

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

**KEY WORDS** r-formula 辅助角公式 • compound 复合角 • double angle 二倍角 • further trigonometry 三角学

## Objective

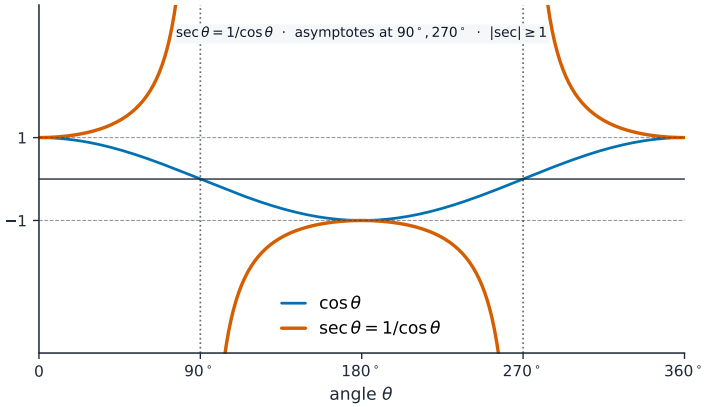
Build the skills to answer exam questions on **further trigonometry** 三角学 — the reciprocal functions, the identities $\sec^2\theta \equiv 1+\tan^2\theta$, **compound** 复合角 and **double angle** 二倍角 formulae, and the **R-formula** 辅助角公式.

**You must be able to:**

- use $\sec^2\theta \equiv 1+\tan^2\theta$ and $\csc^2\theta \equiv 1+\cot^2\theta$
- apply compound- and double-angle formulae
- write $a\sin\theta + b\cos\theta$ as $R\sin(\theta+\alpha)$ and solve

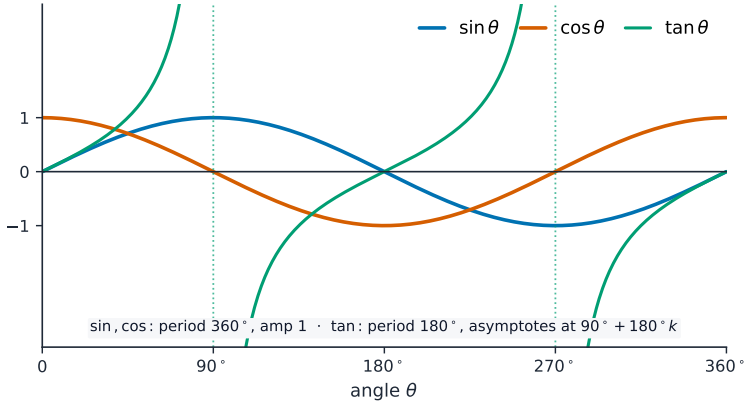
## 1 Worked examples

■ **Reciprocal identity & R-formula**




*$\sec\theta = 1/\cos\theta$ rises to an asymptote wherever $\cos\theta = 0$*






*The underlying sine and cosine graphs*

**Solve** $2\tan^2\theta + 3\sec\theta = 18$ **for** $-180° < \theta < 180°$**.** Use $\tan^2\theta = \sec^2\theta - 1$: $2\sec^2\theta + 3\sec\theta - 20 = 0 \Rightarrow (2\sec\theta - 5)(\sec\theta + 4) = 0$. $\cos\theta = \frac{2}{5}$ or $-\frac{1}{4} \Rightarrow \theta = \pm 66.4°,\ \pm 104.5°$.

**R-formula:** $a\sin\theta + b\cos\theta = R\sin(\theta+\alpha)$, $R = \sqrt{a^2+b^2}$, $\tan\alpha = \frac{b}{a}$.

## 2 Practice

**2.1** Write $\sec^2\theta$ in terms of $\tan\theta$. [1]

**2.2** Use a double-angle formula to write $\sin 2\theta$ in terms of $\sin\theta$ and $\cos\theta$. [1]

## 3 Exam-style questions

**3.1** Expressed as $R\sin(\theta+\alpha)$, the value of $R$ for $3\sin\theta + 4\cos\theta$ is: [1]

- **A** 5
- **B** 7
- **C** $\sqrt{7}$
- **D** 12

**3.2** Express $5\sin\theta - 12\cos\theta$ in the form $R\sin(\theta - \alpha)$ with $R > 0$ and $0° < \alpha < 90°$. [3]

**3.3** (a) Show that $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} \equiv 2\sec^2\theta$. [3]

(b) Hence solve $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = 8$ for $0° \leqslant \theta \leqslant 180°$. [3]



## Solutions

**2.1** $\sec^2\theta = 1 + \tan^2\theta$.

**2.2** $\sin 2\theta = 2\sin\theta\cos\theta$.

**3.1 A**. $R = \sqrt{3^2 + 4^2} = 5$.

**3.2** $R = \sqrt{5^2 + 12^2} = 13$; $\tan\alpha = \frac{12}{5} \Rightarrow \alpha = 67.4°$; $13\sin(\theta - 67.4°)$.

**3.3** (a) common denominator: $\dfrac{(1+\sin\theta)+(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = \dfrac{2}{1-\sin^2\theta} = \dfrac{2}{\cos^2\theta} = 2\sec^2\theta$.
(b) $2\sec^2\theta = 8 \Rightarrow \sec^2\theta = 4 \Rightarrow \cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{2}$; $\theta = 60°$ or $120°$.