

2.2 Logarithmic and exponential functions

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS logarithmic and exponential functions 对数与指数函数
• laws of logarithms 对数定律 • logarithms 对数 • exponential function 指数函数
• linear form 线性形式

Objective

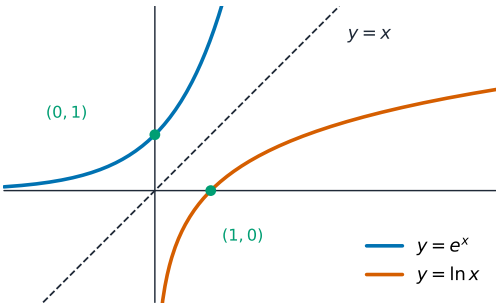
Build the skills to answer exam questions on **logarithmic and exponential functions** 对数与指数函数 — the laws of logs, solving equations with the unknown in the power, and reducing $y = Ax^n$ to **linear form** 线性形式.

You must be able to:

- use the laws of logarithms and $\ln / e^x$ as inverses
- solve equations such as $a^x = b$ by taking logs
- use a log-log (or log-linear) plot to find unknown constants

1 Worked examples

■ Solving and linearising



$\ln$ and $\exp$ inverse • reflect in $y = x$

e^x and $\ln x$ undo each other, so each is the other reflected in $y = x$

Solve $4^x < 0.05$. $x \ln 4 < \ln 0.05 \Rightarrow x < \frac{\ln 0.05}{\ln 4} = -2.16$ (3 s.f.).

Linear form: $y = Ax^n \Rightarrow \ln y = \ln A + n \ln x$ — plot $\ln y$ against $\ln x$: gradient n , intercept $\ln A$.

2 Practice

2.1 Write $\log 12$ in terms of $\log 2$ and $\log 3$. [2]

2.2 Solve $e^{2x} = 7$, giving x to 3 s.f. [2]

3 Exam-style questions

3.1 The solution of $3^x = 20$ is $x =$ [1]

- **A** $\frac{\ln 20}{\ln 3}$
 - **B** $\frac{\ln 3}{\ln 20}$
 - **C** $\ln \frac{20}{3}$
 - **D** $20 - 3$
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3.2 Solve the equation $\ln(2x + 1) + \ln 3 = \ln(4x + 9)$. [3]

3.3 The variables x and y satisfy $y = Ax^n$. When $\ln y$ is plotted against $\ln x$, a straight line of gradient 1.5 passing through $(0, 2)$ is obtained.

(a) Find the values of n and A . [3]

(b) Find y when $x = 4$. [2]

Solutions

2.1 $12 = 2^2 \times 3$, so $\log 12 = 2 \log 2 + \log 3$.

2.2 $2x = \ln 7 \Rightarrow x = \frac{1}{2} \ln 7 = 0.973$.

3.1 A. $x \ln 3 = \ln 20 \Rightarrow x = \frac{\ln 20}{\ln 3}$.

3.2 $\ln(3(2x + 1)) = \ln(4x + 9)$; $6x + 3 = 4x + 9 \Rightarrow 2x = 6$; $x = 3$.

3.3 (a) gradient $= n = 1.5$; intercept $\ln A = 2 \Rightarrow A = e^2 = 7.39$.

(b) $y = 7.39 \times 4^{1.5} = 7.39 \times 8 = 59.1$.