A-LEVEL MATHEMATICS • EXERCISE SHEET

2.1 Algebra (modulus, factor and remainder theorems)

Name: ____________ Class: ________ Date: __________ **Total: 16 marks**

KEY WORDS factor 因式定理 • remainder 余数 • remainder theorem 余数定理 • polynomial 多项式 • modulus 绝对值

Objective

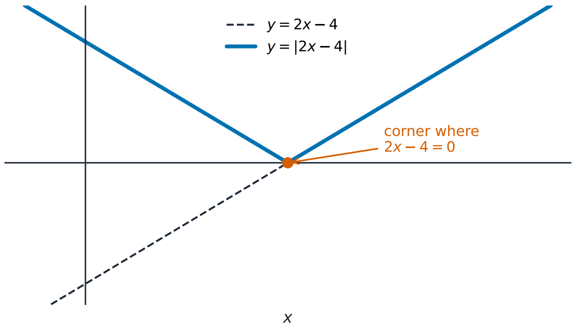
Build the skills to answer exam questions on **algebra** 代数 — the **modulus** 绝对值, and the **factor** 因式定理 and **remainder theorems** 余数定理 for polynomials.

You must be able to:

- solve modulus equations and inequalities ($|x-a|<b$, $|a|=|b| \Leftrightarrow a^2=b^2$)
- use the remainder theorem (remainder $= p(a)$)
- use the factor theorem to factorise a polynomial

1 Worked examples

Modulus & factor theorem



$y=|f(x)|$ reflects parts of $y=f(x)$ below the x-axis above it

Taking the modulus folds the part below the axis up into a "V"

Solve $|3x+8|<9$. $-9<3x+8<9 \Rightarrow -17<3x<1 \Rightarrow -\frac{17}{3}<x<\frac{1}{3}$.

$p(x)=2x^4+kx^3+kx^2+17x+18$ **has factor** $(x+2)$. **Find** k. $p(-2)=0$: $32-8k+4k-34+18=0 \Rightarrow 16-4k=0 \Rightarrow k=4$.

2 Practice

2.1 Solve $|2x-1|=7$. [2]

2.2 Find the remainder when x^3-2x+5 is divided by $(x-2)$. [2]

3 Exam-style questions

3.1 $(x-3)$ is a factor of x^3-4x^2+x+c. The value of c is: [1]

- **A** -6
- **B** 6
- **C** 12
- **D** -12

3.2 Solve the inequality $|x+2|>|x-4|$. [3]

3.3 The polynomial $p(x) = x^3 + ax^2 + bx + 6$ has a factor $(x - 1)$ and leaves a remainder of 8 when divided by $(x + 1)$.

(a) Form two equations in a and b and solve them. [5]

(b) Hence factorise $p(x)$ completely. [3]

Solutions

2.1 $2x - 1 = \pm 7$; $x = 4$ or $x = -3$.

2.2 $p(2) = 8 - 4 + 5 = 9$.

3.1 B. $p(3) = 27 - 36 + 3 + c = 0 \Rightarrow c = 6$.

3.2 square both sides: $(x+2)^2 > (x-4)^2$; $x^2 + 4x + 4 > x^2 - 8x + 16 \Rightarrow 12x > 12$; $x > 1$.

3.3 (a) factor $(x - 1)$: $p(1) = 1 + a + b + 6 = 0 \Rightarrow a + b = -7$; remainder $p(-1) = -1 + a - b + 6 = 8 \Rightarrow a - b = 3$; add the equations: $2a = -4$, so $a = -2$, $b = -5$.
(b) $p(x) = x^3 - 2x^2 - 5x + 6$; divide by $(x-1)$ to get $x^2 - x - 6$; $x^2 - x - 6 = (x-3)(x+2)$; so $p(x) = (x-1)(x-3)(x+2)$.