

2.1 Algebra (modulus, factor and remainder theorems)

Name: _____ Class: _____ Date: _____

Total: 16 marks

Objective

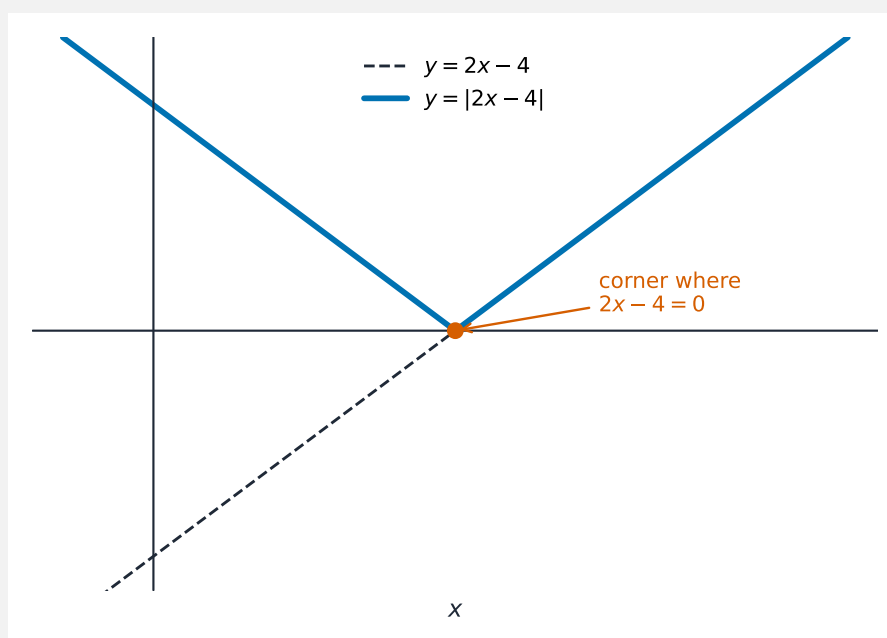
Build the skills to answer exam questions on **algebra** 代数—the **modulus** 绝对值, and the **factor** 因式定理 and **remainder theorems** 余数定理 for polynomials.

You must be able to:

- solve modulus equations and inequalities ($|x - a| < b$, $|a| = |b| \Leftrightarrow a^2 = b^2$)
- use the remainder theorem (remainder = $p(a)$)
- use the factor theorem to factorise a polynomial

1 Worked examples

■ Modulus & factor theorem



Taking the modulus folds the part below the axis up into a "V"

Solve $|3x + 8| < 9$. $-9 < 3x + 8 < 9 \Rightarrow -17 < 3x < 1 \Rightarrow -\frac{17}{3} < x < \frac{1}{3}$.

$p(x) = 2x^4 + kx^3 + kx^2 + 17x + 18$ has factor $(x + 2)$. Find k . $p(-2) = 0$:

$$32 - 8k + 4k - 34 + 18 = 0 \Rightarrow 16 - 4k = 0 \Rightarrow k = 4.$$

2 Practice

2.1 Solve $|2x - 1| = 7$. [2]

2.2 Find the remainder when $x^3 - 2x + 5$ is divided by $(x - 2)$. [2]

3 Exam-style questions

3.1 $(x - 3)$ is a factor of $x^3 - 4x^2 + x + c$. The value of c is: [1]

- A -6
- B 6
- C 12
- D -12

3.2 Solve the inequality $|x + 2| > |x - 4|$. [3]

3.3 The polynomial $p(x) = x^3 + ax^2 + bx + 6$ has a factor $(x - 1)$ and leaves a remainder of 8 when divided by $(x + 1)$.

(a) Form two equations in a and b and solve them. [5]

(b) Hence factorise $p(x)$ completely. [3]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **2.1 Algebra** past-paper sheet in the Library, or try these in **Practice mode**:

- 9709/21 N25 —Q3 (factor/remainder theorem)
- 9709/21 N25 —Q7 (polynomial)
- 9709/22 N25 —Q3 (modulus / polynomial)

Solutions

2.1 $2x - 1 = \pm 7$; $x = 4$ or $x = -3$.

2.2 $p(2) = 8 - 4 + 5 = 9$.

3.1 B. $p(3) = 27 - 36 + 3 + c = 0 \Rightarrow c = 6$.

3.2 square both sides: $(x+2)^2 > (x-4)^2$; $x^2 + 4x + 4 > x^2 - 8x + 16 \Rightarrow 12x > 12$; $x > 1$.

3.3 (a) factor $(x-1)$: $p(1) = 1 + a + b + 6 = 0 \Rightarrow a + b = -7$; remainder $p(-1) = -1 + a - b + 6 = 8 \Rightarrow a - b = 3$; add the equations: $2a = -4$, so $a = -2$, $b = -5$.

(b) $p(x) = x^3 - 2x^2 - 5x + 6$; divide by $(x-1)$ to get $x^2 - x - 6$; $x^2 - x - 6 = (x-3)(x+2)$; so $p(x) = (x-1)(x-3)(x+2)$.