

1.8 Integration

Name: _____ Class: _____ Date: _____

Total: 15 marks

KEY WORDS definite integral 定积分 • volume of revolution 旋转体体积 • integration 积分
• region 区域

Objective

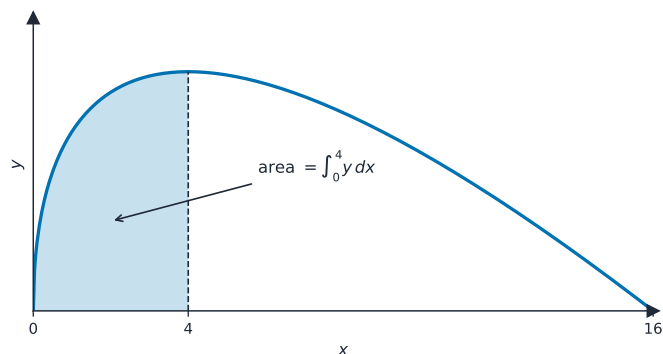
Build the skills to answer exam questions on **integration** 积分 — indefinite and **definite integrals** 定积分, area under a curve, and **volume of revolution** 旋转体体积.

You must be able to:

- integrate powers and $(ax + b)^n$, finding $+C$ from a point
- evaluate a definite integral and find an area
- find a volume of revolution with $V = \pi \int y^2 dx$

1 Worked examples

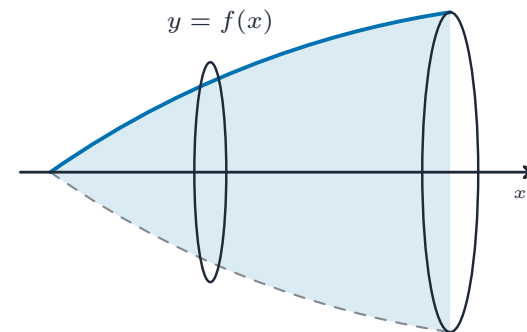
■ Area and volume



$$\int_a^b f(x) dx = \text{net signed area between } y = f(x) \text{ and the } x\text{-axis}$$

The definite integral $\int_p^q y dx$ is the shaded area under the curve

$$\int_0^4 (4x^{1/2} - x) dx = \left[\frac{8}{3}x^{3/2} - \frac{x^2}{2} \right]_0^4 = \frac{8}{3}(8) - 8 = \frac{40}{3}.$$



rotate the region around the x -axis to sweep a solid

Rotating the region about the x -axis: each slice is a disc of area πy^2 , so $V = \pi \int y^2 dx$

2 Practice

2.1 Find $\int (6x^2 - 4x + 1) dx$. [2]

2.2 Evaluate $\int_1^2 \frac{1}{x^2} dx$. [2]

3 Exam-style questions

3.1 $\int_0^3 2x \, dx$ equals: [1]

- A 6
- B 9
- C 12
- D 18

3.2 A curve has $\frac{dy}{dx} = 3x^2 - 4x$ and passes through $(2, 5)$. Find the equation of the curve. [4]

3.3 The region R is bounded by the curve $y = \sqrt{x}$, the x -axis and the line $x = 4$.

(a) Find the area of R . [3]

(b) Find the volume when R is rotated through 360° about the x -axis. [3]

Solutions

2.1 $2x^3 - 2x^2 + x + C$.

2.2 $\left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$.

3.1 B. $[x^2]_0^3 = 9$.

3.2 $y = \int (3x^2 - 4x) \, dx = x^3 - 2x^2 + C$; use $(2, 5)$: $5 = 8 - 8 + C \Rightarrow C = 5$; $y = x^3 - 2x^2 + 5$.

3.3 (a) $\int_0^4 x^{1/2} \, dx = \left[\frac{2}{3}x^{3/2}\right]_0^4 = \frac{2}{3}(8) = \frac{16}{3}$.

(b) $V = \pi \int_0^4 (\sqrt{x})^2 \, dx = \pi \int_0^4 x \, dx = \pi \left[\frac{x^2}{2}\right]_0^4 = 8\pi$.