1.8 Integration

Name: ______________ Class: __________ Date: ____________ **Total: 15 marks**

Objective

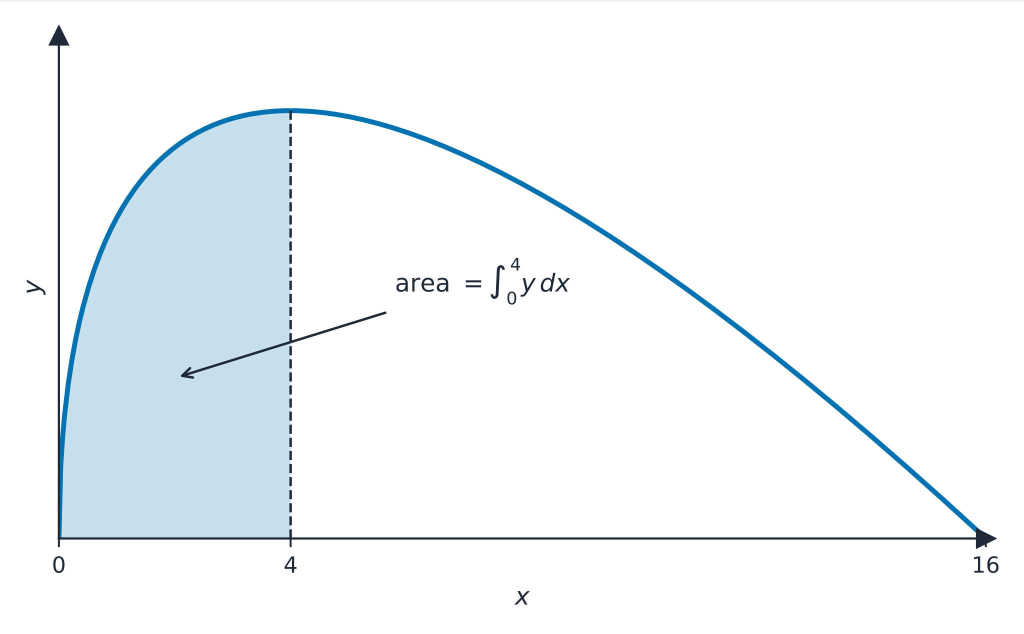
Build the skills to answer exam questions on **integration** 积分—indefinite and **definite integrals** 定积分, area under a curve, and **volume of revolution** 旋转体体积.

You must be able to:

- integrate powers and $(ax+b)^n$, finding $+C$ from a point
- evaluate a definite integral and find an area
- find a volume of revolution with $V = \pi \int y^2\,dx$

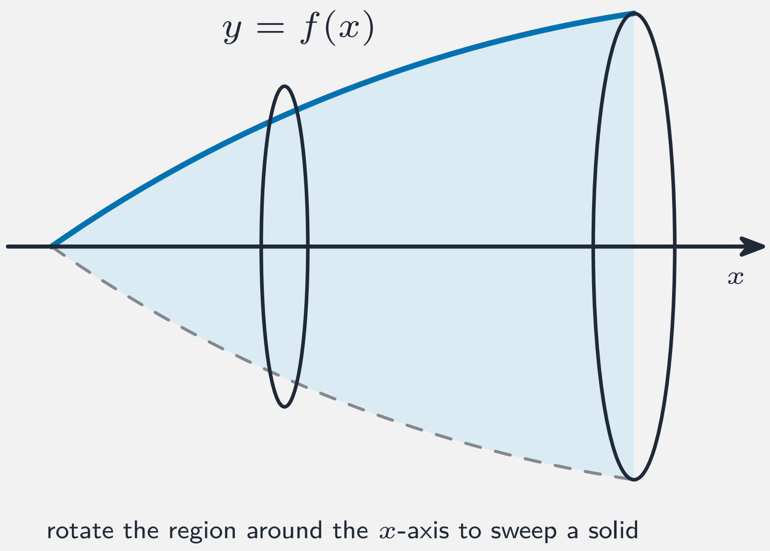
1 Worked examples

Area and volume



The definite integral $\int_p^q y\,dx$ is the shaded area under the curve

$$\int_0^4 (4x^{1/2} - x)\,dx = \left[\tfrac{8}{3}x^{3/2} - \tfrac{x^2}{2}\right]_0^4 = \tfrac{8}{3}(8) - 8 = \tfrac{40}{3}.$$



Rotating the region about the x-axis: each slice is a disc of area πy^2, so $V = \pi \int y^2 \, dx$

2 Practice

2.1 Find $\displaystyle\int (6x^2 - 4x + 1)\, dx$. [2]

2.2 Evaluate $\displaystyle\int_1^2 \frac{1}{x^2}\, dx$. [2]

3 Exam-style questions

3.1 $\displaystyle\int_0^3 2x\, dx$ equals: [1]

- **A** 6
- **B** 9
- **C** 12
- **D** 18

3.2 A curve has $\dfrac{dy}{dx} = 3x^2 - 4x$ and passes through $(2, 5)$. Find the equation of the curve. [4]

3.3 The region R is bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$.

(a) Find the area of R. [3]

(b) Find the volume when R is rotated through 360° about the x-axis. [3]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **1.8 Integration** past-paper sheet in the Library, or try these in **Practice mode**:

- 9709/11 N25 —Q8 (definite integral / area)
- 9709/11 N25 —Q11 (area and volume)
- 9709/12 N25 —Q4 (integration)

Solutions

2.1 $2x^3 - 2x^2 + x + C$.

2.2 $\left[-\dfrac{1}{x}\right]_1^2 = -\dfrac{1}{2} - (-1) = \dfrac{1}{2}$.

3.1 **B**. $[x^2]_0^3 = 9$.

3.2 $y = \displaystyle\int (3x^2 - 4x)\,dx = x^3 - 2x^2 + C$; use $(2, 5)$: $5 = 8 - 8 + C \Rightarrow C = 5$; $y = x^3 - 2x^2 + 5$.

3.3 (a) $\displaystyle\int_0^4 x^{1/2}\,dx = \left[\tfrac{2}{3}x^{3/2}\right]_0^4 = \tfrac{2}{3}(8) = \dfrac{16}{3}$.

(b) $V = \pi \displaystyle\int_0^4 (\sqrt{x})^2\,dx = \pi \int_0^4 x\,dx = \pi \left[\tfrac{x^2}{2}\right]_0^4 = 8\pi$.