

1.6 Series

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS progression 数列 • arithmetic 等差 • geometric progression 等比数列
• sum to infinity 无穷和 • common ratio 公比

Objective

Build the skills to answer exam questions on **series** 级数 — the **binomial expansion** 二项展开式, **arithmetic** 等差 and **geometric progressions** 等比数列, and the **sum to infinity** 无穷和.

You must be able to:

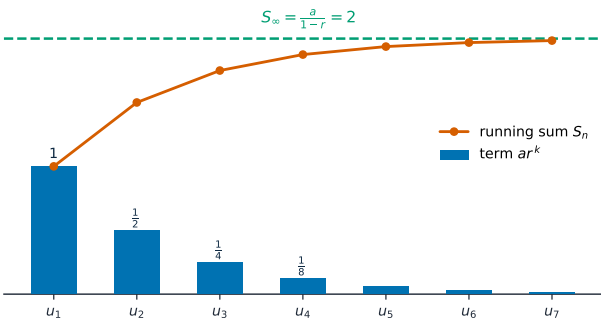
- expand $(a + b)^n$ and find a specific term/coefficient
- use the AP and GP formulas for the n th term and the sum
- use $S_\infty = \frac{a}{1 - r}$ when $|r| < 1$

1 Worked examples

■ Binomial & GP

First three terms of $(2 - px)^5$:

$$2^5 + \binom{5}{1} 2^4(-px) + \binom{5}{2} 2^3(-px)^2 = 32 - 80px + 80p^2x^2.$$



$$S_n = \frac{a(1-r^n)}{1-r} \cdot S_\infty = \frac{a}{1-r} \text{ only if } |r| < 1$$

A convergent GP ($|r| < 1$): the terms shrink and the running sum approaches

$$S_\infty = \frac{a}{1 - r}$$

A GP has third term 18 and $S_3 = 26$, with $r < 0$. From $ar^2 = 18$ and $a(1 + r + r^2) = 26$: $8r^2 - 18r - 18 = 0$, so $r = -\frac{3}{4}$, $a = 32$, and $S_\infty = \frac{32}{1 + \frac{3}{4}} = \frac{128}{7}$.

2 Practice

2.1 Find the 10th term of the AP 3, 7, 11, ... [2]

2.2 State the condition on r for a GP to have a sum to infinity. [1]

3 Exam-style questions

3.1 The coefficient of x^2 in $(1 + 2x)^6$ is: [1]

- **A** 12
- **B** 30
- **C** 60
- **D** 15

3.2 The first term of a GP is 24 and the sum to infinity is 40.

(a) Find the common ratio. [3]

(b) Find the sum of the first three terms. [2]

3.3 An AP has first term 5 and common difference 3. The sum of the first n terms is 440. Find n . [4]

Solutions

2.1 $u_{10} = a + 9d = 3 + 9(4) = 39$.

2.2 $|r| < 1$.

3.1 C. $\binom{6}{2}(2)^2 = 15 \times 4 = 60$.

3.2 (a) $S_{\infty} = \frac{a}{1-r} = 40 \Rightarrow \frac{24}{1-r} = 40 \Rightarrow 1-r = 0.6; r = 0.4$.

(b) $S_3 = \frac{24(1-0.4^3)}{1-0.4} = \frac{24(0.936)}{0.6} = 37.44$.

3.3 $S_n = \frac{n}{2}(2a + (n-1)d) = 440 \Rightarrow \frac{n}{2}(10 + 3(n-1)) = 440; 3n^2 + 7n - 880 = 0;$
 $(3n+55)(n-16) = 0; n = 16$ (reject negative).