

1.5 Trigonometry

Name: _____ Class: _____ Date: _____

Total: 12 marks

KEY WORDS trigonometric equation 三角方程 • sine 正弦 • function 函数 • identity 恒等式
• trigonometry 三角学

Objective

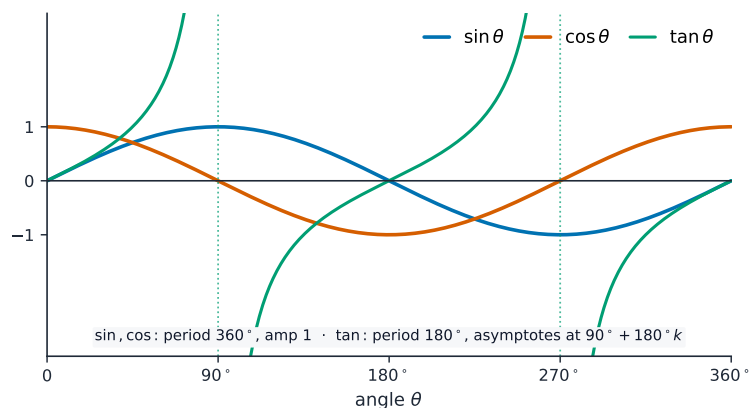
Build the skills to answer exam questions on **trigonometry** 三角学 — exact values, the **identities** 恒等式, and solving **trigonometric equations** 三角方程 in a given interval.

You must be able to:

- recall exact values of $\sin$, $\cos$, $\tan$ for 30° , 45° , 60°
- use $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta \equiv 1$
- solve equations (including quadratics in a trig function) over an interval

1 Worked examples

■ Solving a trig equation



sin and cos stay between -1 and 1 ; tan shoots off at 90° and 270°

Solve $6 \sin \theta = 1 + \frac{2}{\sin \theta}$ for $-180^\circ < \theta < 180^\circ$.

$$\times \sin \theta: 6 \sin^2 \theta - \sin \theta - 2 = 0 \Rightarrow (3 \sin \theta - 2)(2 \sin \theta + 1) = 0. \sin \theta = \frac{2}{3} \Rightarrow \theta = 41.8^\circ, 138.2^\circ; \sin \theta = -\frac{1}{2} \Rightarrow \theta = -30^\circ, -150^\circ.$$

2 Practice

2.1 Write down the exact value of $\cos 30^\circ$. [1]

2.2 Use an identity to write $\frac{\sin \theta}{\cos \theta}$ as a single function. [1]

3 Exam-style questions

3.1 The number of solutions of $\sin \theta = 0.4$ for $0^\circ \leq \theta \leq 360^\circ$ is: [1]

- A 0
- B 1
- C 2
- D 4

3.2 Solve $3 \cos^2 \theta + 2 \sin \theta - 3 = 0$ for $0^\circ \leq \theta \leq 360^\circ$. [5]

3.3 Solve $2 \sin 2x = 1$ for $0^\circ \leq x \leq 180^\circ$.

[4]

Solutions

2.1 $\frac{\sqrt{3}}{2}$.

2.2 $\tan \theta$.

3.1 C. The sine curve gives two solutions in one full turn for $0 < 0.4 < 1$.

3.2 use $\cos^2 \theta = 1 - \sin^2 \theta$: $3(1 - \sin^2 \theta) + 2 \sin \theta - 3 = 0 \Rightarrow -3 \sin^2 \theta + 2 \sin \theta = 0$;
 $\sin \theta (3 \sin \theta - 2) \cdot (-1) = 0 \Rightarrow \sin \theta = 0$ or $\sin \theta = \frac{2}{3}$; $\sin \theta = 0 \Rightarrow \theta = 0^\circ, 180^\circ, 360^\circ$;
 $\sin \theta = \frac{2}{3} \Rightarrow \theta = 41.8^\circ, 138.2^\circ$.

3.3 $\sin 2x = \frac{1}{2}$; for $0^\circ \leq 2x \leq 360^\circ$, $2x = 30^\circ, 150^\circ$; $x = 15^\circ, 75^\circ$.