

1.3 Coordinate geometry

Name: _____ Class: _____ Date: _____

Total: 17 marks**KEY WORDS** circle 圆 • tangent 切线 • straight line 直线 • perpendicular 垂直 • gradient 斜率

Objective

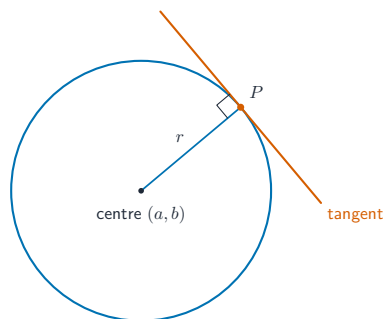
Build the skills to answer exam questions on **coordinate geometry** 坐标几何 — gradients, the equation of a **straight line** 直线, parallel/perpendicular lines, and the equation of a **circle** 圆.

You must be able to:

- find the gradient, midpoint and equation of a line
- use the perpendicular-gradient rule ($m_1 m_2 = -1$)
- find a circle's centre and radius; use the tangent–radius right angle

1 Worked examples

■ Circle and tangent



A tangent touches the circle once and meets the radius at a right angle

$P(1, 1)$ and $Q(7, 11)$ are ends of a diameter. Find the circle's equation.

Centre = midpoint = $(4, 6)$; radius = $\frac{1}{2}\sqrt{6^2 + 10^2} = \frac{1}{2}\sqrt{136} = \sqrt{34}$. So $(x - 4)^2 + (y - 6)^2 = 34$.

Perpendicular: a line of gradient 2 has perpendicular gradient $-\frac{1}{2}$.

2 Practice

2.1 Find the gradient of the line joining $(2, -1)$ and $(6, 7)$. [2]

2.2 Write the centre and radius of $(x - 3)^2 + (y + 2)^2 = 25$. [2]

3 Exam-style questions

3.1 The line through $(0, 4)$ perpendicular to $y = \frac{1}{3}x + 1$ has gradient: [1]

- A $\frac{1}{3}$
- B 3
- C -3
- D $-\frac{1}{3}$

3.2 A circle has equation $x^2 + y^2 + 6x - 10y + 9 = 0$.

(a) Find its centre and radius. [3]

(b) The line $y = 5$ meets the circle at two points. Find their coordinates, and state what this chord is called. [3]

3.3 The points $A(1, 2)$ and $B(5, 10)$ are given.

(a) Find the equation of the perpendicular bisector of AB . [4]

(b) The perpendicular bisector meets the x -axis at C . Find the coordinates of C . [2]

Solutions

2.1 $m = \frac{7 - (-1)}{6 - 2} = \frac{8}{4} = 2.$

2.2 centre $(3, -2)$; radius 5.

3.1 C. Perpendicular gradient $= -1 \div \frac{1}{3} = -3.$

3.2 (a) complete the square: $(x+3)^2 - 9 + (y-5)^2 - 25 + 9 = 0 \Rightarrow (x+3)^2 + (y-5)^2 = 25$; centre $(-3, 5)$, radius 5.

(b) put $y = 5$: $(x+3)^2 + 0 = 25$, so $x+3 = \pm 5$; $x = 2$ or $x = -8$, giving $(2, 5)$ and $(-8, 5)$; $y = 5$ passes through the centre, so the chord is a **diameter** — and its length, 10, is indeed $2r$.

3.3 (a) midpoint $(3, 6)$; gradient $AB = \frac{10 - 2}{5 - 1} = 2$, so bisector gradient $= -\frac{1}{2}$; $y - 6 =$

$-\frac{1}{2}(x - 3) \Rightarrow y = -\frac{1}{2}x + \frac{15}{2}.$

(b) set $y = 0$: $0 = -\frac{1}{2}x + \frac{15}{2} \Rightarrow x = 15$, so $C(15, 0).$