A-LEVEL MATHEMATICS • EXERCISE SHEET

1.1 Quadratics

Name: ____________ Class: ________ Date: __________ **Total: 16 marks**

KEY WORDS quadratic 二次式 • quadratic equation 二次方程 • discriminant 判别式 • simultaneous equations 联立方程 • real roots 实根

Objective

Build the skills to answer exam questions on **quadratics 二次式** — completing the square, the **discriminant 判别式**, solving equations and inequalities, and simultaneous equations.

You must be able to:

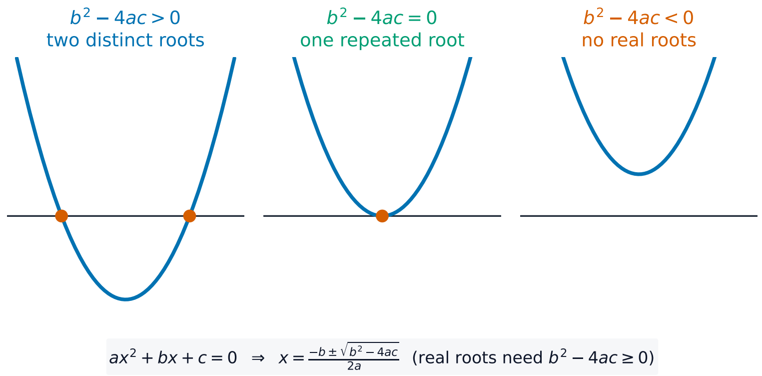
- write $ax^2 + bx + c$ in completed-square form $a(x+p)^2 + q$
- use $b^2 - 4ac$ to decide the number of real roots
- solve quadratic equations and inequalities
- solve a linear–quadratic pair by substitution

1 Worked examples

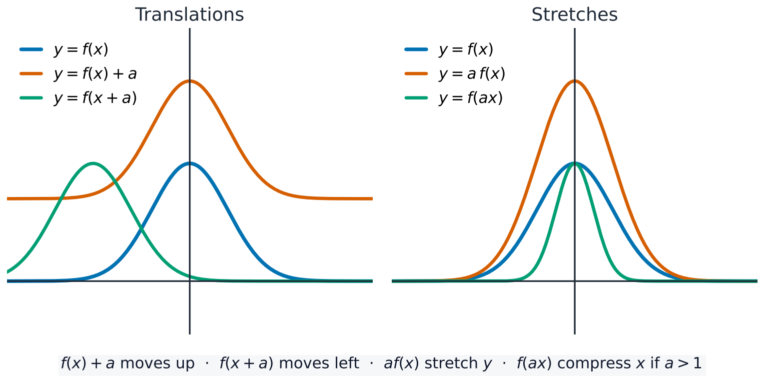
■ Completing the square & the discriminant

Write $2x^2 - 12x + 5$ as $a(x+p)^2 + q$.

$$2x^2 - 12x + 5 = 2(x^2 - 6x) + 5 = 2((x-3)^2 - 9) + 5 = 2(x-3)^2 - 13.$$



The sign of $b^2 - 4ac$ decides how many times the parabola meets the x-axis



Completing the square shifts and stretches the parabola

Find the values of k for which $x^2 + kx + 9 = 0$ has two distinct real roots.

Two distinct roots need $b^2 - 4ac > 0$: $k^2 - 36 > 0 \Rightarrow (k-6)(k+6) > 0 \Rightarrow k < -6$ or $k > 6$.

2 Practice

2.1 Express $x^2 + 8x + 3$ in the form $(x + p)^2 + q$. [2]

2.2 State the condition on the discriminant for a quadratic to have **no** real roots. [1]

3 Exam-style questions

3.1 The equation $2x^2 - 4x + c = 0$ has a repeated root. The value of c is: [1]

- **A** 0
- **B** 1
- **C** 2
- **D** 4

3.2 Express $3x^2 + 12x + 7$ in the form $a(x + b)^2 + c$ and hence write down the least value of the expression. [4]

3.3 Find the set of values of k for which the line $y = kx - 2$ meets the curve $y = x^2 - 3x + 2$ at two distinct points. [4]

3.4 Solve the simultaneous equations $y = 2x - 1$ and $x^2 + y^2 = 2$. [4]

Solutions

2.1 $(x+4)^2 - 13$.

2.2 $b^2 - 4ac < 0$.

3.1 **C**. Repeated root needs $b^2 - 4ac = 0$: $16 - 8c = 0 \Rightarrow c = 2$.

3.2 $3(x^2+4x)+7 = 3((x+2)^2-4)+7 = 3(x+2)^2-5$; least value $= -5$ (when $x = -2$).

3.3 set equal: $x^2 - 3x + 2 = kx - 2 \Rightarrow x^2 - (3+k)x + 4 = 0$; two points need $b^2 - 4ac > 0$: $(3+k)^2 - 16 > 0$; $(k+7)(k-1) > 0$; $k < -7$ or $k > 1$.

3.4 substitute: $x^2 + (2x-1)^2 = 2 \Rightarrow 5x^2 - 4x - 1 = 0$; $(5x+1)(x-1) = 0 \Rightarrow x = -\frac{1}{5}$ or $x = 1$; $y = 2x - 1$ gives $(1, 1)$ and $\left(-\frac{1}{5}, -\frac{7}{5}\right)$.