Exercise walk-through

Hypothesis tests ▪ 6.5

eXercise

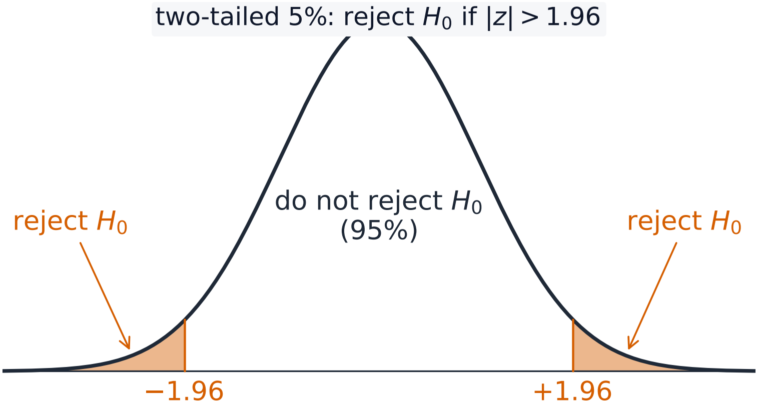
You must be able to

- state H_0, H_1 and choose one- or two-tailed
- compute a z test statistic and compare with the critical value
- explain Type I and Type II errors

Method — A two-tailed test

Claim $\mu = 50$**,** $\sigma = 8$**,** $n = 64$**,** $\bar{x} = 52$**, test at** 5%**:** $H_0 : \mu = 50,\ H_1 : \mu \neq 50.$
$z = \dfrac{52 - 50}{8/8} = 2.$ Since $2 > 1.96$, **reject** H_0.

Method — A two-tailed test



A standard normal curve with both tails beyond ±1.96 shaded

Practice 2.1 ▪ 1 mark

State the meaning of a **Type I error**.

Solution 2.1

Worked steps

rejecting H_0 when it is in fact true **B1**.

Practice 2.2 ▪ 1 mark

Give the critical z value for a one-tailed test at the 5% level.

Solution 2.2

Method: A two-tailed test

Worked steps

1.645 **B1**.

Exam-style 3.1 ▪ 1 mark

In a test at the 5% level, H_0 is rejected when the p-value is:

A greater than 0.05

B less than 0.05

C equal to 1

D negative

Solution 3.1

Method: A two-tailed test

A greater than 0.05

B less than 0.05 ✓

C equal to 1

D negative

Worked steps

B. Reject H_0 when the p-value is below the significance level 0.05.

Exam-style 3.2 ▪ 2 marks

A machine should fill bottles to a mean of 500 ml with $\sigma = 6$ ml. A sample of 36 bottles has mean $\bar{x} = 502$ ml. Test at the 5% level whether the mean has increased.

(a) State H_0 and H_1, and whether the test is one- or two-tailed.

(b) Carry out the test and state your conclusion.

Solution 3.2

Method: A two-tailed test

Worked steps

(a) $H_0 : \mu = 500$, $H_1 : \mu > 500$ **M1**; one-tailed **A1**.

(b) $z = \dfrac{502 - 500}{6/\sqrt{36}} = \dfrac{2}{1} = 2$ **M1** **M1**; critical value 1.645; since $2 > 1.645$ **M1**, reject H_0: there is evidence the mean has increased **A1**.

Exam-style 3.3 ▪ 3 marks

A coin is tossed 20 times to test $H_0 : p = 0.5$ against $H_1 : p > 0.5$ at the 5% level, using the number of heads $X \sim B(20, 0.5)$. The critical region is $X \geqslant 15$.
(a) Find $P(X \geqslant 15)$ under H_0 and explain why $X \geqslant 15$ is used.
(b) State what a Type I error would mean in this context.

Solution 3.3

Method: A two-tailed test

Worked steps

(a) $P(X \geqslant 15) = 0.0207$ (from tables) **M1**; this is the smallest "tail" region with probability $\leqslant 0.05$, so it is the 5% critical region **M1** **A1**.
(b) concluding the coin is biased towards heads ($p > 0.5$) when in fact it is fair **B1**.