Exercise walk-through

Sampling and estimation ▪ 6.4

eXercise

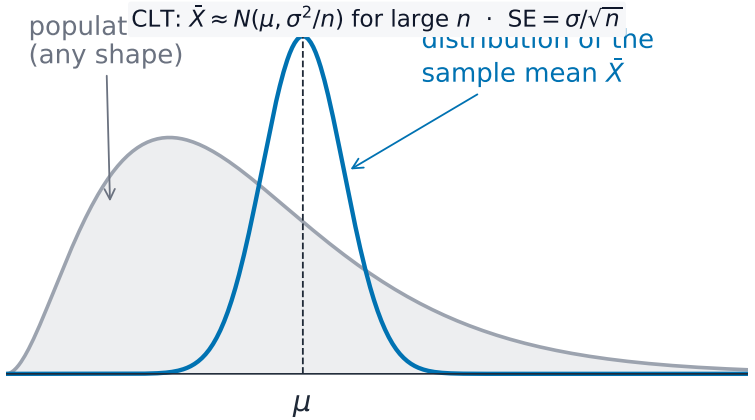
You must be able to

- use $E(\bar{X}) = \mu$, $\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$ and the CLT
- find unbiased estimates of the population mean and variance
- find a confidence interval $\bar{x} \pm z\dfrac{\sigma}{\sqrt{n}}$

Method — Confidence interval

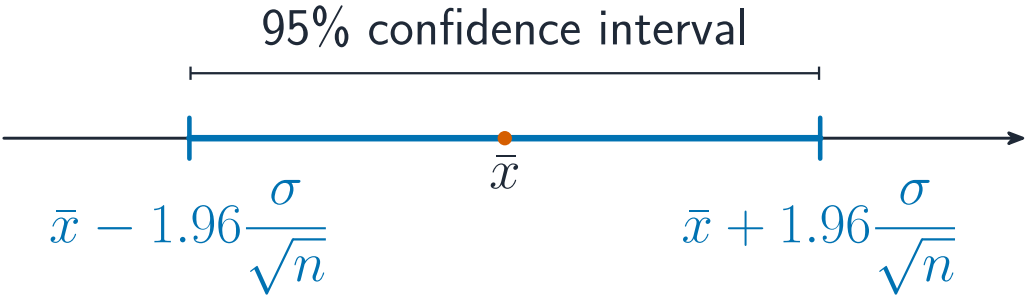
$n = 64$, $\bar{x} = 50$, $\sigma = 8$: $50 \pm 1.96\dfrac{8}{8} = 50 \pm 1.96 = (48.0,\ 52.0)$.

Method — Confidence interval



A skewed population and the narrow near-normal bell of the sample mean

Method — Confidence interval



A number line with the interval reaching each side of the sample mean

Practice 2.1 ▪ 2 marks

A population has $\sigma = 10$. For a sample of $n = 25$, find the standard error $\dfrac{\sigma}{\sqrt{n}}$.

Solution 2.1

Method: Confidence interval

Worked steps

$$\frac{10}{\sqrt{25}} = \frac{10}{5} = 2 \quad \text{M1} \; \text{A1}.$$

Practice 2.2 ▪ 1 mark

State the value of z used in a 95% confidence interval.

Solution 2.2

Method: Confidence interval

Worked steps

1.96 B1.

Exam-style 3.1 ▪ 1 mark

As the sample size n increases, $\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$:

A increases

B decreases

C stays the same

D becomes negative

Solution 3.1

Method: Confidence interval

A increases

B decreases ✓

C stays the same

D becomes negative

Worked steps

B. Dividing by a larger n makes $\mathrm{Var}(\bar{X})$ smaller.

Exam-style 3.2 ▪ 3 marks

A random sample of 100 light bulbs has mean lifetime $\bar{x} = 1200$ hours. The population standard deviation is $\sigma = 150$ hours.

(a) Find a 95% confidence interval for the mean lifetime.

(b) State what changes to the interval if a 99% level ($z = 2.576$) is used instead.

Solution 3.2

Method: Confidence interval

Worked steps

(a) $1200 \pm 1.96\dfrac{150}{\sqrt{100}} = 1200 \pm 1.96(15) = 1200 \pm 29.4$ M1 M1; $(1170.6,\ 1229.4)$ A1.

(b) the interval becomes **wider** (a higher confidence level needs a larger z, here 2.576) M1 A1.

Exam-style 3.3 ▪ 4 marks

A sample of 8 values gives $\sum x = 120$ and $\sum x^2 = 1840$. Find unbiased estimates of the population mean and variance.

Solution 3.3

Method: Confidence interval

Worked steps

mean $\bar{x} = \dfrac{120}{8} = 15$ M1 A1; unbiased variance $s^2 = \dfrac{1}{n-1}\left(\sum x^2 - \dfrac{(\sum x)^2}{n}\right) = \dfrac{1}{7}\left(1840 - \dfrac{120^2}{8}\right) = \dfrac{1}{7}(1840 - 1800) = \dfrac{40}{7} = 5.71$ M1 A1.