Past-paper walk-through

Continuous random variables ▪ 6.3

eXercise

Questions in this set

- 9709/61 N25 Q7 ▪ 7 marks
- 9709/62 N25 Q5 ▪ 9 marks
- 9709/61 J25 Q7 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 7

9709/61 N25 ▪ Q7 ▪ 7 marks

The time, in minutes, taken by students to complete a test is modelled by the random variable X with probability density function $f(x) = -\frac{3}{4}(x-3)(x-5)$ for $3 \leqslant x \leqslant 5$, and $f(x) = 0$ otherwise.

(a) Find the probability that a randomly chosen student takes longer than 4.5 minutes to complete the test. **[4]**

(b) Write down the median of X. **[1]**

(c) Without performing an integration, use your answer to part (a) to find $P(3.5 < X < 4.5)$. **[2]**

Solution 7

(a)

Mark scheme

- $P(X > 4.5) = \int_{4.5}^{5} -\frac{3}{4}(x-3)(x-5)\,dx = 0.15625 = \frac{5}{32}$.

Solution 7

(b)

Mark scheme

- By symmetry of f about $x = 4$, the median is 4.

Solution 7

(c)

Mark scheme

- By symmetry, $P(X < 3.5) = P(X > 4.5) = \frac{5}{32}$, so $P(3.5 < X < 4.5) = 1 - 2 \times \frac{5}{32} = \frac{11}{16} = 0.6875$.

Question 5

9709/62 N25 ▪ Q5 ▪ 9 marks

A random variable X has probability density function given by $f(x) = k(2x^2 - x^3)$ for $0 \leqslant x \leqslant 2$, and $f(x) = 0$ otherwise.

(a) Show that $k = \frac{3}{4}$. **[3]**

(b)(i) Write down the value of $P(X \leqslant m)$. **[1]**

(b)(ii) Hence find $P(E(X) \leqslant X \leqslant m)$. **[5]**

Solution 5

(a)

Mark scheme

- $\int_0^2 k(2x^2 - x^3)\,dx = k\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2 = k \cdot \frac{4}{3} = 1 \Rightarrow k = \frac{3}{4}$. AG.

Solution 5

(b)(i)

Mark scheme

- $P(X \leqslant m) = 0.5$.

(b)(ii)

Mark scheme

- $E(X) = \int_0^2 x \cdot \frac{3}{4}(2x^2 - x^3)\,dx = 1.2$; $P(X \leqslant 1.2) = \frac{3}{4}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^{1.2} = 0.4752$.
  So $P(1.2 \leqslant X \leqslant m) = 0.5 - 0.4752 = 0.0248$.

Question 7

9709/61 J25 ▪ Q7 ▪ 8 marks

X is a random variable with probability density function given by $f(x) = 1 + \cos \pi x$ for $0 \leqslant x \leqslant 1$, and $f(x) = 0$ otherwise.

(a) Show that $\mathrm{P}\left(X < \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{\pi}$. **[3]**

(b) Show that $\mathrm{E}(X) = \frac{1}{2} - \frac{2}{\pi^2}$. **[5]**

Solution 7

(a)

Mark scheme

- $\mathrm{P}\left(X < \frac{1}{2}\right) = \int_0^{1/2} (1 + \cos \pi x)\, dx = \left[x + \frac{\sin \pi x}{\pi}\right]_0^{1/2} = \frac{1}{2} + \frac{\sin(\pi/2)}{\pi} = \frac{1}{2} + \frac{1}{\pi}$ (AG).

Solution 7

(b) Mark scheme

- $\mathrm{E}(X) = \int_0^1 x(1+\cos\pi x)\,dx = \left[\frac{x^2}{2} + \frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_0^1 = \frac{1}{2} + \frac{\cos\pi}{\pi^2} - \frac{1}{\pi^2} = \frac{1}{2} - \frac{2}{\pi^2}$ (AG).