Exercise walk-through

Linear combinations of random variables ▪ 6.2

eXercise

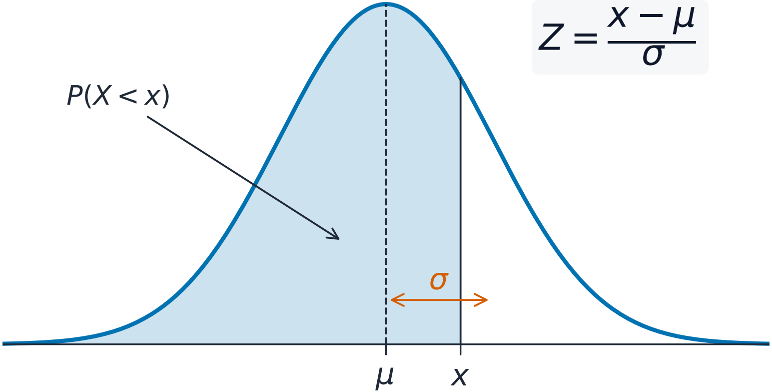
You must be able to

- use $E(aX+b) = aE(X)+b$, $\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$
- use $E(aX+bY) = aE(X)+bE(Y)$, $\mathrm{Var}(aX+bY) = a^2\mathrm{Var}(X)+b^2\mathrm{Var}(Y)$
- recognise that a linear combination of independent normals is normal

Method — Mean and variance rules

X **has mean** 5**, variance** 4**:** $E(3X-1)=3(5)-1=14$; $\mathrm{Var}(3X-1)=3^2(4)=36$.
Watch the signs: $\mathrm{Var}(X-Y)=\mathrm{Var}(X)+\mathrm{Var}(Y)$ (variances **add**).

Method — Mean and variance rules



A normal bell curve — a linear transform of a normal stays normal

Practice 2.1 ▪ 2 marks

X has mean 10 and variance 9. Find $E(2X+5)$.

Solution 2.1

Method: Mean and variance rules

Worked steps

$E(2X + 5) = 2(10) + 5 = 25$ M1 A1.

Practice 2.2 ▪ 1 mark

For independent X, Y with $\mathrm{Var}(X) = 4$, $\mathrm{Var}(Y) = 9$, find $\mathrm{Var}(X + Y)$.

Solution 2.2

Method: Mean and variance rules

Worked steps

$\mathrm{Var}(X + Y) = 4 + 9 = 13$ **B1**.

Exam-style 3.1 ▪ 1 mark

X has variance 5. Then $\mathrm{Var}(4X)$ is:

- **A** 20
- **B** 80
- **C** 9
- **D** 5

Solution 3.1

Method: Mean and variance rules

A 20

B 80 ✓

C 9

D 5

Worked steps

B. $\mathrm{Var}(4X) = 4^2 \times 5 = 80.$

Exam-style 3.2 ▪ 3 marks

Independent variables have $X \sim N(20, 3^2)$ and $Y \sim N(15, 4^2)$.
(a) Find the distribution of $X - Y$.
(b) Find $P(X - Y > 2)$.

Solution 3.2

Method: Mean and variance rules

Worked steps

(a) mean $= 20 - 15 = 5$; variance $= 3^2 + 4^2 = 25$; so $X - Y \sim N(5, 25)$ **M1** **M1** **A1**.

(b) $Z = \dfrac{2-5}{5} = -0.6$; $P(X - Y > 2) = P(Z > -0.6) = \Phi(0.6) = 0.7257$ **M1** **M1** **A1**.

Exam-style 3.3 ▪ 4 marks

The mass of a tin is $X \sim N(500, 4^2)$ g. Four tins are packed in a box of mass $N(100, 2^2)$ g, all independent. Find the mean and standard deviation of the total mass of a packed box.

Solution 3.3

Method: Mean and variance rules

Worked steps

total $T = X_1 + X_2 + X_3 + X_4 + B$; mean $= 4(500) + 100 = 2100$ g **M1** **A1**; variance $= 4(4^2) + 2^2 = 64 + 4 = 68$, so SD $= \sqrt{68} = 8.25$ g **M1** **A1**.