

Exercise walk-through

The Poisson distribution ■ 6.1

eXercise

You must be able to

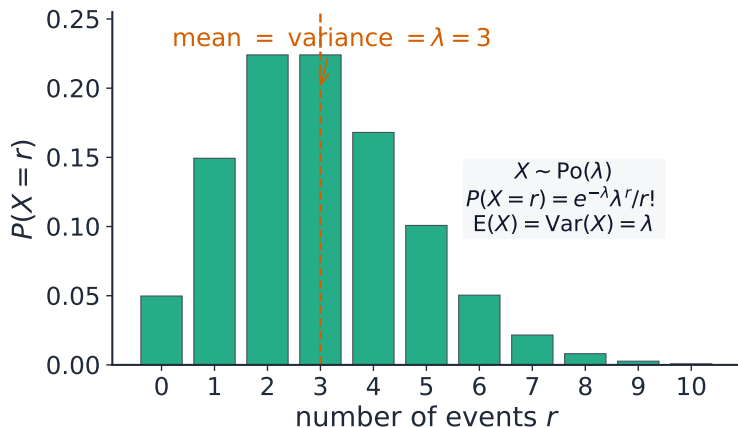
- use $P(X = r) = e^{-\lambda} \frac{\lambda^r}{r!}$ (mean = variance = λ)
- combine independent Poisson variables (the sum is Poisson)
- use $\text{Po}(np)$ to approximate $B(n, p)$ when n large, p small

Method — Poisson probabilities

$$X \sim \text{Po}(3): P(X=2) = e^{-3} \frac{3^2}{2!} = 4.5e^{-3} = 0.224.$$

$$P(X \geq 1) = 1 - P(X=0) = 1 - e^{-3} = 0.950.$$

Method — Poisson probabilities



A bar chart of $\text{Po}(3)$, leaning right

Practice 2.1 ■ 2 marks

$X \sim \text{Po}(2)$. Find $P(X = 0)$.

Solution 2.1

Worked steps

$$P(X = 0) = e^{-2} = 0.135 \quad \text{M1} \quad \text{A1}.$$

Practice 2.2 ■ 1 mark

State the variance of $\text{Po}(4.5)$.

Solution 2.2

Method: Poisson probabilities

Worked steps

variance = $\lambda = 4.5$ **B1**.

Exam-style 3.1 ■ 1 mark

For $X \sim \text{Po}(\lambda)$, the mean equals:

A λ^2

B $\sqrt{\lambda}$

C λ

D $\frac{1}{\lambda}$

Solution 3.1

Method: Poisson probabilities

A λ^2

B $\sqrt{\lambda}$

C λ



D $\frac{1}{\lambda}$

Worked steps

C. For a Poisson distribution the mean is λ .

Exam-style 3.2 ■ 2 marks

Calls arrive at a switchboard at a mean rate of 1.5 per minute, as a Poisson process.

- (a) Find the probability of exactly 2 calls in a given minute.
- (b) Find the probability of at least 1 call in a 2-minute period.

Solution 3.2

Method: Poisson probabilities

Worked steps

$$(a) P(X = 2) = e^{-1.5} \frac{1.5^2}{2!} = e^{-1.5}(1.125) = 0.251 \quad \text{M1 A1}.$$

$$(b) \text{ over 2 minutes } \lambda = 3; P(X \geq 1) = 1 - e^{-3} = 1 - 0.0498 = 0.950 \quad \text{M1 M1 A1}.$$

Exam-style 3.3 ■ 4 marks

A factory makes items with a 0.5% defect rate. A box holds 400 items. Using a Poisson approximation, find the probability that a box contains exactly 3 defective items.

Solution 3.3

Worked steps

$$\lambda = np = 400 \times 0.005 = 2 \quad \text{M1}; \quad P(X = 3) = e^{-2} \frac{2^3}{3!} = e^{-2} \frac{8}{6} \quad \text{M1 M1}; = 0.1353 \times 1.333 = 0.180 \quad \text{A1}.$$