

Exercise walk-through

The normal distribution ■ 5.5

eXercise

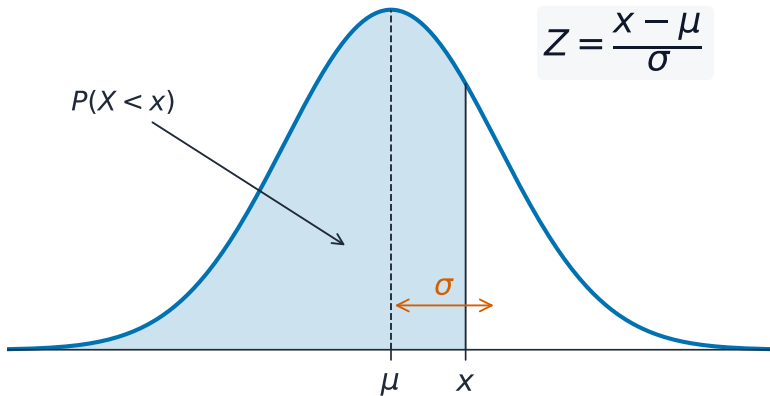
You must be able to

- standardise with $Z = \frac{X - \mu}{\sigma}$ and read Φ
- find an unknown mean or standard deviation from a given probability
- use the normal approximation with a **continuity correction** 连续性校正

Method — Standardise & find μ

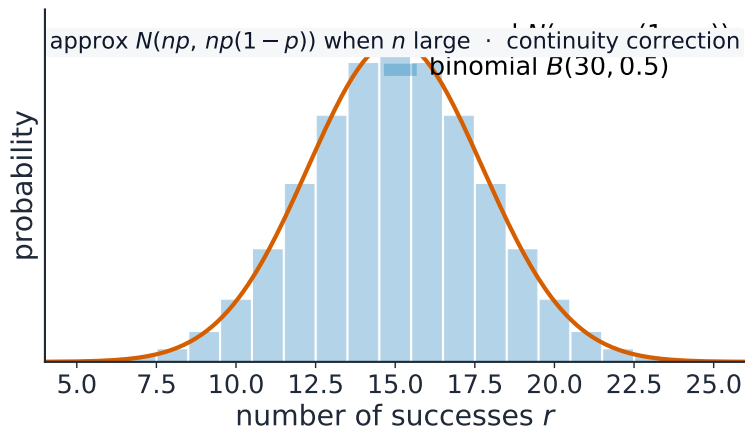
$$X \sim N(\mu, 0.14^2), P(X < 1.48) = 0.22. \Phi^{-1}(0.22) = -0.772, \text{ so}$$
$$\frac{1.48 - \mu}{0.14} = -0.772 \Rightarrow \mu = 1.59.$$

Method — Standardise & find μ



A symmetric bell curve with the area up to x shaded

Method — Standardise & find μ



Binomial bars with a matching normal curve over them

Practice 2.1 ■ 2 marks

$X \sim N(50, 4^2)$. Find the Z value for $X = 56$.

Solution 2.1

Worked steps

$$Z = \frac{56 - 50}{4} = 1.5 \quad \text{M1} \quad \text{A1}.$$

Practice 2.2 ■ 1 mark

State the continuity correction used for $P(X \geq 20)$ in a normal approximation.

Solution 2.2

Method: Standardise & find μ

Worked steps

use $P(X \geq 19.5)$ **B1**.

Exam-style 3.1 ■ 1 mark

$X \sim N(0, 1)$. Using $\Phi(1) = 0.8413$, $P(Z > 1)$ is:

A 0.8413

B 0.1587

C 0.3413

D 0.5

Solution 3.1

A 0.8413

B 0.1587



C 0.3413

D 0.5

Worked steps

B. $P(Z > 1) = 1 - 0.8413 = 0.1587.$

Exam-style 3.2 ■ 3 marks

The heights of plants are $X \sim N(82, 5^2)$ cm.

(a) Find $P(X > 90)$.

(b) Find the height exceeded by the tallest 10% of plants.

Solution 3.2

Worked steps

(a) $Z = \frac{90 - 82}{5} = 1.6$ **M1**; $P(X > 90) = 1 - \Phi(1.6) = 1 - 0.9452 = 0.0548$

M1 A1.

(b) top 10%: $\Phi(z) = 0.90 \Rightarrow z = 1.282$ **M1**; $x = 82 + 1.282(5) = 88.4$ cm

M1 A1.

Exam-style 3.3 ■ 5 marks

A fair coin is tossed 100 times. Using a normal approximation, find the probability of obtaining at least 60 heads.

Solution 3.3

Method: Standardise & find μ

Worked steps

$X \sim B(100, 0.5) \approx N(50, 25)$ (M1); continuity correction $P(X \geq 60) \rightarrow P(Y > 59.5)$
 (M1); $Z = \frac{59.5 - 50}{5} = 1.9$ (M1); $P = 1 - \Phi(1.9) = 1 - 0.9713 = 0.0287$ (M1) (A1).