Past-paper walk-through

Discrete random variables ▪ 5.4

eXercise

Questions in this set

- 9709/51 N25 Q1 ▪ 6 marks
- 9709/51 N25 Q5 ▪ 8 marks
- 9709/52 N25 Q1 ▪ 3 marks
- 9709/52 N25 Q2 ▪ 7 marks
- 9709/52 N25 Q6 ▪ 9 marks
- 9709/51 J25 Q4 ▪ 12 marks
- 9709/51 J25 Q6 ▪ 9 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

9709/51 N25 ▪ Q1 ▪ 6 marks

The random variable X takes the value x with probability kx^2, where k is a constant and x takes the values -2, 1, 2, 3 only.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. **[3]**

(b) Find $\mathrm{E}(X)$. **[1]**

(c) Find $\mathrm{P}(X \neq 2 \mid X > 0)$. **[2]**

Solution 1

(a)

Mark scheme

- $4k + k + 4k + 9k = 1 \Rightarrow k = \frac{1}{18}$. Table: $\mathrm{P}(X = -2) = \frac{4}{18}$, $\mathrm{P}(X = 1) = \frac{1}{18}$, $\mathrm{P}(X = 2) = \frac{4}{18}$, $\mathrm{P}(X = 3) = \frac{9}{18}$.

Solution 1

(b)

Mark scheme

- $\mathrm{E}(X) = 28k = \frac{28}{18} = \frac{14}{9} \approx 1.56.$

Solution 1

(c) Mark scheme

- $P(X \neq 2 \mid X > 0) = \dfrac{\frac{1}{18} + \frac{9}{18}}{\frac{1}{18} + \frac{4}{18} + \frac{9}{18}} = \dfrac{10}{14} = \frac{5}{7} \approx 0.714.$

Question 5

9709/51 N25 ▪ Q5 ▪ 8 marks

On any given day, Cooper either wears a blue jumper or a green jumper or does not wear a jumper. The probability that he wears a blue jumper is 0.6 and the probability that he wears a green jumper is 0.3. His choice on any day is independent of his choice on any other day.

(a) Find the probability that, in a week (7 days), Cooper wears a blue jumper on at least 5 days. **[3]**

(b) Use a suitable approximation to find the probability that, in any 150-day period, Cooper does not wear a jumper on fewer than 22 days. **[5]**

Solution 5

(a)

Mark scheme

- $X \sim B(7, 0.6)$: $\binom{7}{5}(0.6)^5(0.4)^2 + \binom{7}{6}(0.6)^6(0.4) + (0.6)^7 = 0.420.$

Solution 5

(b)

Mark scheme

- $\mathrm{P}(\text{no jumper}) = 0.1$; $n = 150$ gives mean 15, variance 13.5. Normal approx with continuity correction:
  $\mathrm{P}(X < 22) = \mathrm{P}\left(Z < \frac{21.5-15}{\sqrt{13.5}}\right) = \mathrm{P}(Z < 1.769) = 0.962.$

Question 1

9709/52 N25 ▪ Q1 ▪ 3 marks

A coin is biased so that the probability of obtaining a head when it is thrown is 0.4. The coin is thrown repeatedly until the first head is obtained.

(a) Find the probability that the first head is obtained on the 5th throw. **[1]**

(b) Find the probability that the first head is obtained after the 6th throw. **[2]**

Solution 1

(a)

Mark scheme

- Geometric: $P(X = 5) = (0.6)^4(0.4) = 0.0518.$

Solution 1

(b)

Mark scheme

- $P(X > 6) = (0.6)^6 = 0.0467.$

Question 2

9709/52 N25 ▪ Q2 ▪ 7 marks

Kayla has a bag containing 3 red marbles, 1 blue marble and 2 green marbles. She selects one marble from the bag at random and does not replace it. She repeats this process until she obtains a green marble. The random variable X is the number of marbles she needs to select until she obtains a green marble.

(a) Draw up the probability distribution table for X. **[4]**

(b) Find $\mathrm{Var}(X)$. **[3]**

Solution 2

(a)

Mark scheme

- $X = 1, 2, 3, 4, 5$ with $P = \frac{1}{3},\ \frac{4}{15},\ \frac{1}{5},\ \frac{2}{15},\ \frac{1}{15}$ (4 non-green, 2 green, no replacement).

Solution 2

(b)

Mark scheme

- $E(X) = \frac{7}{3}$ and $E(X^2) = 7$, so $\mathrm{Var}(X) = 7 - \left(\frac{7}{3}\right)^2 = \frac{14}{9} \approx 1.56$.

Question 6

9709/52 N25 ▪ Q6 ▪ 9 marks

For a randomly chosen person, their next birthday is equally likely to occur on any day of the week, independently of any other person's birthday.

(a) Find the probability that, out of 10 randomly chosen people, none of them will have their next birthday on a Saturday or Sunday. **[1]**

(b) Find the probability that, out of 10 randomly chosen people, fewer than 3 will have their next birthday on a Wednesday. **[3]**

(c) Use a suitable approximation to find the probability that, out of 392 randomly chosen people, more than 65 will have their next birthday on a Friday. **[5]**

Solution 6

(a)

Mark scheme

- $P(\text{neither Sat nor Sun}) = \frac{5}{7}$, so $\left(\frac{5}{7}\right)^{10} = 0.0346.$

Solution 6

(b)

Mark scheme

- $X \sim B(10, \frac{1}{7})$: $P(X < 3) = \left(\frac{6}{7}\right)^{10} + 10 \cdot \frac{1}{7} \left(\frac{6}{7}\right)^{9} + 45 \left(\frac{1}{7}\right)^{2} \left(\frac{6}{7}\right)^{8} = 0.838.$

Solution 6

(c)

Mark scheme

- $X \sim B(392, \frac{1}{7})$, approximated by $N(56, 48)$.
  $P(X > 65) \approx P\left(Z > \frac{65.5-56}{\sqrt{48}}\right) = P(Z > 1.371) = 0.0852.$

Question 4

9709/51 J25 ▪ Q4 ▪ 12 marks

Every Saturday, a particular community holds a 'Puzzle' event to raise money for a new Leisure Centre. Competitors attempt to solve a puzzle as quickly as possible. Last Saturday, 600 competitors took part. The times taken to complete the puzzle were normally distributed with mean 32.4 minutes and standard deviation 2.5 minutes.

(a) How many competitors would you expect to have times within 1.2 minutes of the mean time? **[4]**

(b) In this Saturday's event, 60% of the competitors had times less than 36.0 minutes. 9 competitors who took part are selected at random. Find the probability that at least 2 and fewer than 8 of these competitors had times less than 36.0 minutes. **[3]**

(c) 80 competitors who took part in this Saturday's event are selected at random. Use a suitable approximation to find the probability that more than 50 of these competitors had times less than 36.0 minutes. **[5]**

Solution 4

(a)

Mark scheme

- $\mathrm{P}\left(-\frac{1.2}{2.5} < Z < \frac{1.2}{2.5}\right) = 2\Phi(0.48) - 1 = 0.3688$; expected number $= 0.3688 \times 600 \approx 221.$

Solution 4

(b)

Mark scheme

- $X \sim B(9, 0.6)$: $\mathrm{P}(2 \leqslant X < 8) = 1 - \mathrm{P}(X = 0, 1, 8, 9) = 0.926.$

Solution 4

(c)

Mark scheme

- $X \sim B(80, 0.6)$, mean 48, variance 19.2. Normal approximation with continuity correction: $\mathrm{P}(X > 50) = \mathrm{P}\left(Z > \frac{50.5-48}{\sqrt{19.2}}\right) = \mathrm{P}(Z > 0.571) = 0.284$.

Question 6

9709/51 J25 ▪ Q6 ▪ 9 marks

A bag contains 10 marbles, of which 4 are red and 6 are blue. Four marbles are selected from the bag at random, without replacement. The random variable X denotes the number of blue marbles selected.

(a) Show that $\mathrm{P}(X = 2) = \frac{3}{7}$. **[2]**

(b) Draw up the probability distribution table for X. **[4]**

(c) Find the probability that at least 2 of the marbles chosen are blue, given that at least 1 red marble and at least 1 blue marble are chosen. **[3]**

Solution 6

(a)

Mark scheme

- $\mathrm{P}(X = 2) = \dfrac{\binom{6}{2}\binom{4}{2}}{\binom{10}{4}} = \dfrac{15 \times 6}{210} = \dfrac{3}{7}$ (AG).

Solution 6

(b)

Mark scheme

- $\mathrm{P}(X=0)=\frac{1}{210}$, $\mathrm{P}(X=1)=\frac{4}{35}$, $\mathrm{P}(X=2)=\frac{3}{7}$, $\mathrm{P}(X=3)=\frac{8}{21}$, $\mathrm{P}(X=4)=\frac{1}{14}$.

Solution 6

(c) Mark scheme

- $$\frac{\mathrm{P}(X=2)+\mathrm{P}(X=3)}{\mathrm{P}(X=1)+\mathrm{P}(X=2)+\mathrm{P}(X=3)}=\frac{\frac{3}{7}+\frac{8}{21}}{\frac{4}{35}+\frac{3}{7}+\frac{8}{21}}=\frac{17/21}{97/105}=\frac{85}{97}=0.876.$$