

Exercise walk-through

Discrete random variables ■ 5.4

eXercise

You must be able to

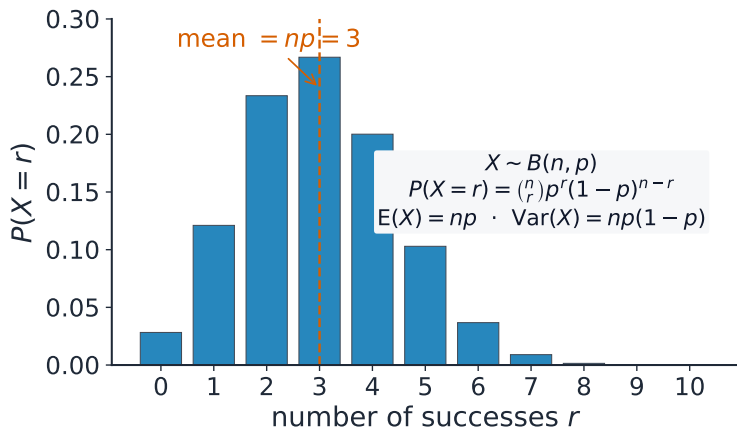
- use a distribution table (probabilities sum to 1); find $E(X)$ and $\text{Var}(X)$
- use $B(n, p)$: $P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$, $E(X) = np$
- use the geometric model $P(X = r) = (1 - p)^{r-1} p$, $E(X) = \frac{1}{p}$

Method — Binomial distribution

$X \sim B(10, 0.3)$: $P(X = 2) = \binom{10}{2}(0.3)^2(0.7)^8 = 0.233$; $E(X) = np = 3$.

Distribution: $E(X) = \sum xP(X=x)$, $\text{Var}(X) = \sum x^2P(X=x) - [E(X)]^2$.

Method — Binomial distribution



A bar chart of $B(10, 0.3)$ clustered around the mean 3

Practice 2.1 ■ 1 mark

$X \sim B(8, 0.25)$. Find $E(X)$.

Solution 2.1

Worked steps

$$E(X) = np = 8 \times 0.25 = 2 \text{ (B1).}$$

Practice 2.2 ■ 1 mark

A geometric variable has $p = 0.2$. Find $E(X)$.

Solution 2.2

Worked steps

$$E(X) = \frac{1}{p} = \frac{1}{0.2} = 5 \text{ (B1)}.$$

Exam-style 3.1 ■ 1 mark

For $X \sim B(12, 0.5)$, the variance $\text{Var}(X)$ is:

A 6

B 3

C 12

D 1.5

Solution 3.1

A 6

B 3



C 12

D 1.5

Worked steps

B. $\text{Var}(X) = np(1 - p) = 12(0.5)(0.5) = 3.$

Exam-style 3.2 ■ 1 mark

A discrete random variable X has the distribution:

x	1	2	3	4
$P(X = x)$	0.1	0.3	k	0.2

- (a) Find k .
(b) Find $E(X)$ and $\text{Var}(X)$.

Solution 3.2

Method: Binomial distribution

Worked steps

(a) probabilities sum to 1: $0.1 + 0.3 + k + 0.2 = 1 \Rightarrow k = 0.4$ **B1**.

(b) $E(X) = 1(0.1) + 2(0.3) + 3(0.4) + 4(0.2) = 0.1 + 0.6 + 1.2 + 0.8 = 2.7$ **M1 A1**;

$E(X^2) = 1(0.1) + 4(0.3) + 9(0.4) + 16(0.2) = 0.1 + 1.2 + 3.6 + 3.2 = 8.1$ **M1**;

$\text{Var}(X) = 8.1 - 2.7^2 = 8.1 - 7.29 = 0.81$ **M1 A1**.

Exam-style 3.3 ■ 3 marks

A fair coin is tossed 6 times. Find the probability of obtaining exactly 4 heads.

Solution 3.3

Worked steps

$$X \sim B(6, 0.5); P(X = 4) = \binom{6}{4}(0.5)^6 = 15 \times \frac{1}{64} = \frac{15}{64} = 0.234 \quad \text{M1 M1 A1}.$$