Past-paper walk-through

Permutations and combinations ▪ 5.2

eXercise

Questions in this set

- 9709/51 N25 Q7 ▪ 10 marks
- 9709/52 N25 Q7 ▪ 10 marks
- 9709/51 J25 Q2 ▪ 7 marks
- 9709/51 J25 Q5 ▪ 7 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 7

9709/51 N25 ▪ Q7 ▪ 10 marks

(a) How many different arrangements are there of the 10 letters in the word SEYCHELLES? **[1]**

(b) How many different arrangements are there of the 10 letters in the word SEYCHELLES in which there are exactly two letters between the Ss and one of these two letters is C? **[3]**

(c) How many different arrangements are there of the 10 letters in the word SEYCHELLES in which there is an S at the beginning, an S at the end and the three Es are not all next to each other? **[3]**

(d) 5 letters are selected at random from the 10 letters in the word SEYCHELLES. Find the probability that these 5 letters include the three Es. **[3]**

Solution 7

(a)

Mark scheme

- $\dfrac{10!}{2!\,3!\,2!} = 151\,200$ (S$\times 2$, E$\times 3$, L$\times 2$).

Solution 7

(b)

Mark scheme

- With S, (one letter), C as the gap pattern: $\dfrac{7!}{2!\,3!} \times 2 \times 7 = 5\,880$.

Solution 7

(c)

Mark scheme

- Ss fixed at the two ends: $\dfrac{8!}{2!\,3!} - \dfrac{6!}{2!} = 3360 - 360 = 3\,000$ (total minus those with the three Es together).

Solution 7

(d)

Mark scheme

- $\dfrac{\binom{7}{2}}{\binom{10}{5}} = \dfrac{21}{252} = \frac{1}{12} \approx 0.0833.$

Question 7

9709/52 N25 ▪ Q7 ▪ 10 marks

(a) Find the number of different arrangements of the 10 letters in the word ZOOLOGICAL in which the three Os are together and the two Ls are **not** next to each other. **[4]**

(b) Find the number of different arrangements of the 10 letters in the word ZOOLOGICAL in which there are exactly 5 letters between the two Ls. **[3]**

(c) Two letters are chosen at random from the 10 letters in the word ZOOLOGICAL. Find the probability that these two letters are different. **[3]**

Solution 7

(a)

Mark scheme

- Treat OOO as one block (8 items including two Ls): $\frac{8!}{2!} = 20160$; subtract those with the Ls together ($7! = 5040$): $20160 - 5040 = 15120$.

Solution 7

(b)

Mark scheme

- The Ls fill positions $(i, i+6)$ for $i = 1, 2, 3, 4$ (4 ways); the remaining 8 letters (incl. 3 Os) arrange in $\frac{8!}{3!} = 6720$ ways: $4 \times 6720 = 26880$.

Solution 7

(c)

Mark scheme

- $\binom{10}{2} = 45$ pairs; identical-letter pairs $= \binom{3}{2} + \binom{2}{2} = 4$;
  $P(\text{different}) = \dfrac{45 - 4}{45} = \dfrac{41}{45} \approx 0.911.$

Question 2

9709/51 J25 ▪ Q2 ▪ 7 marks

(a) Find the number of different arrangements of the 8 letters in the word KANGAROO in which the two As are together and the two Os are not together. **[3]**

(b) A fair 8-sided dice has faces labelled K, A, N, G, A, R, O, O. The dice is rolled repeatedly. Find the probability that fewer than 6 rolls of this dice are required to obtain an A. **[2]**

(c) Find the probability that the second A is obtained on the 6th roll of the dice. **[2]**

Solution 2

(a)

Mark scheme

- As together: $\dfrac{7!}{2!} = 2520$ (the two Os among the 7 items); As together and Os together: $6! = 720$. So $2520 - 720 = 1800$.

Solution 2

(b)

Mark scheme

- $\mathrm{P}(A) = \frac{1}{4}$; $\mathrm{P}(\text{fewer than 6 rolls}) = 1 - \left(\frac{3}{4}\right)^5 = \dfrac{781}{1024} = 0.763.$

Solution 2

(c)

Mark scheme

- $\binom{5}{1}\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 \times \frac{1}{4} = \left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4 \times 5 = \dfrac{405}{4096} = 0.0989.$

Question 5

9709/51 J25 ▪ Q5 ▪ 7 marks

In a group of 20 musicians, there are 9 guitarists, 6 pianists and 5 drummers. 6 musicians are selected from these 20 to perform at a concert.

(a) Find the number of different ways in which the 6 musicians can be selected if there must be at least 3 guitarists, at most 2 pianists and exactly 1 drummer. **[4]**

(b) Three bands will be selected from the original group of 20 musicians. Each band will consist of 3 guitarists, 1 pianist and 1 drummer. No musician can be in more than one band. The first band selected will play in France, the second in Italy and the third in Spain. Find the number of different ways in which these three bands can be selected. **[3]**

Solution 5

(a)

Mark scheme

- $3G2P1D: \binom{9}{3}\binom{6}{2}\binom{5}{1} = 6300$; $4G1P1D: \binom{9}{4}\binom{6}{1}\binom{5}{1} = 3780$; $5G0P1D: \binom{9}{5}\binom{5}{1} = 630$. Total $= 10\,710$.

Solution 5

(b)

Mark scheme

- Band 1: $\binom{9}{3}\binom{6}{1}\binom{5}{1} = 2520$; band 2: $\binom{6}{3}\binom{5}{1}\binom{4}{1} = 400$; band 3: $\binom{3}{3}\binom{4}{1}\binom{3}{1} = 12$. Total $= 2520 \times 400 \times 12 = 12\,096\,000$.