

Exercise walk-through

Permutations and combinations ■ 5.2

eXercise

You must be able to

- use nP_r for arrangements and nC_r for selections
- arrange objects with repeats (divide by each repeat factorial)
- count with restrictions (e.g. items together / not together)

Method — Arrangements & selections

Arrangements of NEEDLESS (8 letters, E×3, S×2): $\frac{8!}{3! 2!} = 3360$.

Choosing 3 from 10 (order doesn't matter): ${}^{10}C_3 = \frac{10!}{3! 7!} = 120$.

Method — Arrangements & selections



An assortment of dice — counting equally likely outcomes

Arranging 3 of 4 objects (4P_3)

$$\boxed{4} \times \boxed{3} \times \boxed{2} = 24$$

1st pick 2nd pick 3rd pick

Choosing 3 of 4 (order ignored): ${}^4C_3 = \frac{{}^4P_3}{3!} = 4$

${}^nP_r = n! / (n - r)!$ · ${}^nC_r = {}^nP_r / r!$

Practice 2.1 ■ 1 mark

Evaluate 7P_3 .

Solution 2.1

Worked steps

$${}^7P_3 = 7 \times 6 \times 5 = 210 \quad \text{B1}.$$

Practice 2.2 ■ 1 mark

Evaluate 8C_2 .

Solution 2.2

Worked steps

$${}^8C_2 = \frac{8 \times 7}{2} = 28 \quad \text{B1}.$$

Exam-style 3.1 ■ 1 mark

The number of ways to arrange the letters of BANANA is:

A 720

B 120

C 60

D 360

Solution 3.1

Method: Arrangements & selections

A 720

B 120

C 60



D 360

Worked steps

C. BANANA has 6 letters with A×3, N×2: $\frac{6!}{3!2!} = \frac{720}{12} = 60$.

Exam-style 3.2 ■ 2 marks

A committee of 4 is chosen from 7 women and 5 men.

- (a) Find the number of possible committees.
- (b) Find the number that contain exactly 2 women and 2 men.

Solution 3.2

Worked steps

$$(a) {}^{12}C_4 = \frac{12 \cdot 11 \cdot 10 \cdot 9}{24} = 495 \quad \text{M1 A1}.$$

$$(b) {}^7C_2 \times {}^5C_2 = 21 \times 10 = 210 \quad \text{M1 M1 A1}.$$

Exam-style 3.3 ■ 3 marks

Find the number of arrangements of the 7 letters of PICTURE in which the three vowels (I, U, E) are all together.

Solution 3.3

Method: Arrangements & selections

Worked steps

treat (IUE) as one block $\rightarrow$ 5 items arrange in $5! = 120$ ways **M1**; the vowels arrange among themselves in $3! = 6$ ways **M1**; total $= 120 \times 6 = 720$ **A1**.