Past-paper walk-through

Complex numbers ▪ 3.9

eXercise

Questions in this set

- 9709/31 N25 Q5 ▪ 6 marks
- 9709/31 N25 Q6 ▪ 5 marks
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Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

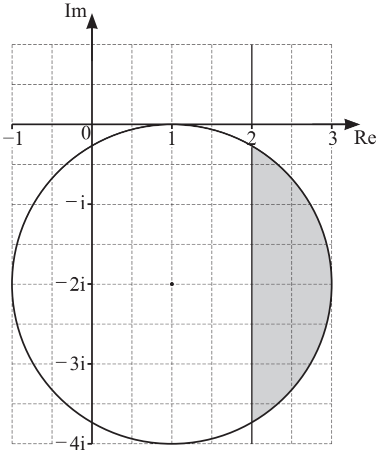
9709/31 N25 ▪ Q5 ▪ 6 marks

The shaded region on the Argand diagram shows points representing complex numbers z defined by two inequalities. The shaded region is bounded by a circle and a line parallel to the imaginary axis. The boundaries of the region are included in the shaded region.

(a) Find two inequalities in terms of z that define the shaded region. **[3]**

(b) Calculate the least value of $\arg z$ for points in this region. **[3]**

Question 5



Solution 5

(a)

Mark scheme

- The circle has centre $1-2i$ and radius 2, and the line is $\mathrm{Re}(z)=2$. The region is $|z-1+2i|\leqslant 2$ and $\mathrm{Re}(z)\geqslant 2$.

Solution 5

(b)

Mark scheme

- The least (most negative) $\arg z$ occurs at the corner where $\mathrm{Re}(z) = 2$ meets the lower circle, $z = 2 - (2 + \sqrt{3})i$. $\arg z = -\tan^{-1}\dfrac{2+\sqrt{3}}{2} \approx -1.08$ rad (-61.8).

Question 6

9709/31 N25 ▪ Q6 ▪ 5 marks

Solve the quadratic equation $(2+i)w^2 + 4w + 2 - i = 0$. Give your answers in the form $x + iy$, where x and y are real. **[5]**

Solution 6

Mark scheme

- Discriminant $= 16 - 4(2+i)(2-i) = 16 - 20 = -4$, $\sqrt{} = 2i$. $w = \dfrac{-4 \pm 2i}{2(2+i)}$, giving $w = -1$ and $w = -\frac{3}{5} + \frac{4}{5}i$.

Question 3

9709/31 J25 ▪ Q3 ▪ 6 marks

Find the complex numbers z for which $\dfrac{z+5i}{z-5}$ is real and $|z| = \sqrt{17}$. Give your answers in the form $z = x + iy$, where x and y are real. **[6]**

Solution 3

Mark scheme

- $\dfrac{z+5i}{z-5}$ real $\Rightarrow x - y = 5$; with $|z|^2 = 17$:
  $x^2 + (x-5)^2 = 17 \Rightarrow x^2 - 5x + 4 = 0$. So $z = 4 - i$ and $z = 1 - 4i$.

Question 6

9709/31 J25 ▪ Q6 ▪ 6 marks

It is given that $z_1 = 3e^{\frac{1}{4}\pi i}$, $z_2 = \frac{3}{2}e^{\frac{5}{6}\pi i}$ and $\omega = 2e^{\frac{1}{2}\pi i}$.

(a) State the values of ωz_1 and ωz_2. Give your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. **[2]**

(b) On a sketch of an Argand diagram with origin O, show the points A, B, C and D representing the complex numbers z_1, z_2, ωz_1 and ωz_2 respectively. **[2]**

(c) State the geometric effects of multiplying z_1 and z_2 by ω. **[2]**

Solution 6

(a)

Mark scheme

- $\omega z_1 = 6e^{\frac{3}{4}\pi i}$; $\omega z_2 = 3e^{\frac{4}{3}\pi i} = 3e^{-\frac{2}{3}\pi i}$.

Solution 6

(b)

Mark scheme

- Plot A (z_1: modulus 3, arg $\frac{1}{4}\pi$), B (z_2: modulus $\frac{3}{2}$, arg $\frac{5}{6}\pi$), C (ωz_1: modulus 6, arg $\frac{3}{4}\pi$), D (ωz_2: modulus 3, arg $-\frac{2}{3}\pi$).

Solution 6

(c)

Mark scheme

- Multiplying by ω is a rotation through $\frac{1}{2}\pi$ (anticlockwise) about O together with an enlargement of scale factor 2.

Question 3

9709/32 J25 ▪ Q3 ▪ 5 marks

On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 3i| \leqslant 2$ and $\frac{1}{4}\pi \leqslant \arg(z - 1 - 2i) \leqslant \frac{3}{4}\pi$. **[5]**

Solution 3

Mark scheme

- Shade the intersection of the disc $|z - 3i| \leqslant 2$ (centre $(0, 3)$, radius 2) with the wedge from $(1, 2)$ where $\arg(z - 1 - 2i)$ lies between $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (region between the two half-lines of gradient ± 1 from $(1, 2)$, opening upward).

Question 5

9709/32 J25 ▪ Q5 ▪ 5 marks

The square roots of $-1 - 4\sqrt{5}\,i$ can be expressed in the Cartesian form $x + iy$, where x and y are real and exact. By first forming a quartic equation in x or y, find the square roots of $-1 - 4\sqrt{5}\,i$ in exact Cartesian form. **[5]**

Solution 5

Mark scheme

- $(x+iy)^2 = -1 - 4\sqrt{5}\,i \Rightarrow x^2 - y^2 = -1$ and $2xy = -4\sqrt{5}$. Eliminating y: $x^4 + x^2 - 20 = 0 \Rightarrow x^2 = 4$. The square roots are $2 - \sqrt{5}\,i$ and $-2 + \sqrt{5}\,i$.