

Exercise walk-through

Complex numbers ■ 3.9

eXercise

You must be able to

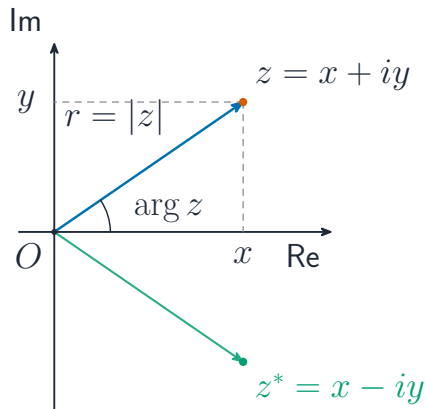
- add, multiply and divide complex numbers (divide using the conjugate)
- find modulus and argument; convert to polar form
- sketch loci such as $|z - a| = r$, $|z - a| = |z - b|$, $\arg(z - a) = \theta$

Method — Division & loci

$|z|$ is the distance from O , $\arg z$ the angle, z^* the reflection in the real axis

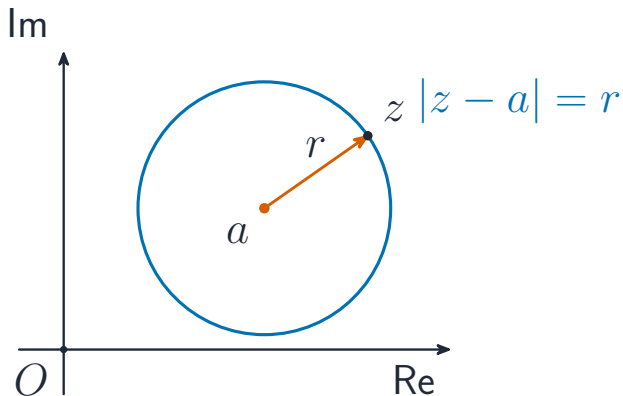
$$\frac{3+i}{1-i} = \frac{(3+i)(1+i)}{2} = \frac{2+4i}{2} = 1+2i.$$

Method — Division & loci



A complex number on the Argand plane with its modulus, argument and conjugate

Method — Division & loci



A circle on the Argand plane: $|z-a|=r$

Practice 2.1 ■ 1 mark

Find the modulus of $z = 3 + 4i$.

Solution 2.1

Worked steps

$$|z| = \sqrt{3^2 + 4^2} = 5 \quad \text{B1}.$$

Practice 2.2 ■ 2 marks

Write $(2 + i)(3 - i)$ in the form $x + iy$.

Solution 2.2

Worked steps

$$(2 + i)(3 - i) = 6 - 2i + 3i - i^2 = 7 + i \quad \text{M1} \quad \text{A1}.$$

Exam-style 3.1 ■ 1 mark

The argument of $z = -1 + i$ (in radians, $-\pi < \arg z \leq \pi$) is:

A $\frac{\pi}{4}$

B $\frac{3\pi}{4}$

C $-\frac{\pi}{4}$

D $\frac{\pi}{2}$

Solution 3.1

A $\frac{\pi}{4}$

B $\frac{3\pi}{4}$



C $-\frac{\pi}{4}$

D $\frac{\pi}{2}$

Worked steps

B. The point $(-1, 1)$ is in the second quadrant: $\arg z = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

Exam-style 3.2 ■ 3 marks

The complex number z satisfies $z = \frac{10}{2 - i}$.

- (a) Express z in the form $x + iy$.
- (b) Find $|z|$ and $\arg z$ (in radians).

Solution 3.2

Method: Division & loci

Worked steps

$$(a) \ z = \frac{10(2+i)}{(2-i)(2+i)} = \frac{20+10i}{5} = 4+2i \quad \text{M1 M1 A1}.$$

$$(b) \ |z| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5} \quad \text{M1}; \arg z = \tan^{-1} \frac{2}{4} = 0.4636 \text{ rad} \quad \text{M1 A1}.$$

Exam-style 3.3 ■ 2 marks

On a single Argand diagram, sketch the locus of points z for which:

(a) $|z - (2 + i)| = 2$.

(b) Describe the locus $|z - 3| = |z + 1|$.

Solution 3.3

Method: Division & loci

Worked steps

- (a) a circle of radius 2 centred at the point $(2, 1)$ **M1** **A1**.
- (b) the perpendicular bisector of the segment joining 3 and -1 — i.e. the vertical line $x = 1$ **M1** **A1**.