Past-paper walk-through

Differential equations ▪ 3.8

eXercise

Questions in this set

- 9709/31 N25 Q8 ▪ 8 marks
- 9709/31 J25 Q10 ▪ 8 marks
- 9709/32 J25 Q8 ▪ 7 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 8

9709/31 N25 ▪ Q8 ▪ 8 marks

The variables x and y satisfy the differential equation $(x^2+1)\dfrac{dy}{dx} = kxe^{2y}$, where k is a constant. It is given that $y = 0$ when $x = 0$ and that $y = -\frac{1}{2}$ when $x = 1$. Solve the differential equation and find the exact value of y when $x = \sqrt{3}$. **[8]**

Solution 8

Mark scheme

- Separating: $-\frac{1}{2}e^{-2y} = \frac{k}{2}\ln(x^2+1) + C$. Using $(0,0)$: $C = -\frac{1}{2}$; using $(1, -\frac{1}{2})$: $k = \dfrac{1-e}{\ln 2}$. At $x = \sqrt{3}$ $(\ln 4 = 2\ln 2)$: $-\frac{1}{2}e^{-2y} = (1-e) - \frac{1}{2} \Rightarrow e^{-2y} = 2e - 1$, so $y = -\frac{1}{2}\ln(2e-1)$.

Question 10

9709/31 J25 ▪ Q10 ▪ 8 marks

(a) Find the quotient and remainder when $x^3 + 5x^2 - 2x - 15$ is divided by $x^2 - 3$. **[3]**

(b) The variables x and y satisfy the differential equation $\dfrac{dy}{dx} = \dfrac{x^3 + 5x^2 - 2x - 15}{6y(x^2 - 3)}$. It is given that $y = 2$ when $x = 2$. Solve the differential equation to obtain an expression for y^2 in terms of x. **[5]**

Solution 10

(a)

Mark scheme

- $x^3 + 5x^2 - 2x - 15 = (x^2 - 3)(x + 5) + x$; quotient $x + 5$, remainder x.

Solution 10

(b)

Mark scheme

- $\int 6y\,dy = \int \left(x + 5 + \dfrac{x}{x^2 - 3}\right) dx \Rightarrow 3y^2 = \frac{1}{2}x^2 + 5x + \frac{1}{2}\ln(x^2 - 3) + C$;
  $y = 2, x = 2 \Rightarrow C = 0$. So $y^2 = \frac{1}{6}x^2 + \frac{5}{3}x + \frac{1}{6}\ln(x^2 - 3)$.

Question 8

9709/32 J25 ▪ Q8 ▪ 7 marks

The variables x and θ satisfy the differential equation $\sin 2\theta \dfrac{dx}{d\theta} = (4x + 3)\cos 2\theta$, and $x = 0$ when $\theta = \frac{1}{12}\pi$. Solve the differential equation and obtain an expression for x in terms of θ. **[7]**

Solution 8

Mark scheme

- Separating variables: $\frac{1}{4}\ln(4x+3) = \frac{1}{2}\ln(\sin 2\theta) + C$. Using $x = 0$ at $\theta = \frac{\pi}{12}$ gives $4x + 3 = 12\sin^2 2\theta$, i.e. $x = 3\sin^2 2\theta - \frac{3}{4} = \frac{3}{4} - \frac{3}{2}\cos 4\theta$.