Past-paper walk-through

Vectors ▪ 3.7

eXercise

Questions in this set

- 9709/31 N25 Q11 ▪ 10 marks
- 9709/31 J25 Q8 ▪ 9 marks
- 9709/32 J25 Q9 ▪ 9 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 11

9709/31 N25 ▪ Q11 ▪ 10 marks

With respect to the origin O, the points A, B, C, D have position vectors $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$, $\overrightarrow{OD} = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}$. The line m passes through A and B.

(a) Find a vector equation for m. **[2]**

(b) Find the position vector of the point of intersection of m and the line passing through the points C and D. **[4]**

(c) Find the position vector of the foot of the perpendicular from C to m. **[4]**

Solution 11

(a)

Mark scheme

- $\overrightarrow{AB} = \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$, so $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + s \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$.

Solution 11

(b)

Mark scheme

- Line CD: $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$. Equating with m gives $t = -2$, $s = 4$; the point of intersection is $(-3, 1, -5)$.

Solution 11

(c)

Mark scheme

- Foot $F = A + s\overrightarrow{AB}$ with $(F - C) \cdot \overrightarrow{AB} = 0 \Rightarrow 6s - 12 = 0 \Rightarrow s = 2$. So $F = (-1, 3, -1)$.

Question 8

9709/31 J25 ▪ Q8 ▪ 9 marks

With respect to the origin O, the points A and B have position vectors $2\mathbf{i} + 4\mathbf{k}$ and $5\mathbf{i} + \mathbf{j} + 6\mathbf{k}$ respectively. The line l_1 passes through the points A and B.

(a) Find a vector equation for the line l_1. **[2]**

(b) The line l_2 has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$. Show that l_1 and l_2 do not intersect. **[4]**

(c) Find the acute angle between the directions of l_1 and l_2. **[3]**

Solution 8

(a)

Mark scheme

- $\overrightarrow{AB} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, so $\mathbf{r} = (2\mathbf{i} + 4\mathbf{k}) + \lambda(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

Solution 8

(b)

Mark scheme

- Equating components of l_1 and l_2: $\lambda = -1$, $\mu = -1$ satisfy two equations but not the third $(1 \neq -1)$, so the lines do not intersect.

Solution 8

(c)

Mark scheme

- $\cos\theta = \dfrac{(3\mathbf{i}+\mathbf{j}+2\mathbf{k})\cdot(\mathbf{i}+2\mathbf{j}+3\mathbf{k})}{\sqrt{14}\sqrt{14}} = \dfrac{11}{14}$, so the acute angle $\theta = 38.2$ (0.667 rad).

Question 9

9709/32 J25 ▪ Q9 ▪ 9 marks

With respect to the origin O, the points A, B and C have position vectors given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$.

(a) Find a vector equation for the line through A and B. **[2]**

(b) Using a scalar product, find the exact value of $\cos BAC$. **[4]**

(c) Hence find the exact area of triangle ABC. **[3]**

Solution 9

(a)

Mark scheme

- $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix}$, so $\mathbf{r} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} + t \begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix}$.

Solution 9

(b)

Mark scheme

- $\overrightarrow{AB} = (-3, 5, 1)$, $\overrightarrow{AC} = (1, 7, 3)$; $\overrightarrow{AB} \cdot \overrightarrow{AC} = 35$, $|\overrightarrow{AB}| = \sqrt{35}$, $|\overrightarrow{AC}| = \sqrt{59}$.
  $\cos BAC = \dfrac{35}{\sqrt{35}\sqrt{59}} = \dfrac{\sqrt{2065}}{59}$.

Solution 9

(c) Mark scheme

- $\sin BAC = \sqrt{1 - \frac{35}{59}} = \sqrt{\frac{24}{59}}$. Area $= \frac{1}{2}\sqrt{35}\,\sqrt{59}\,\sqrt{\frac{24}{59}} = \frac{1}{2}\sqrt{35 \cdot 24} = \sqrt{210}$.