

Exercise walk-through

Vectors ■ 3.7

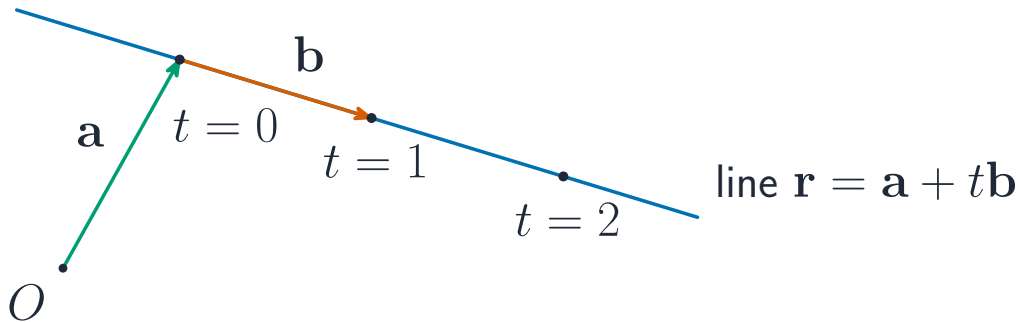
eXercise

You must be able to

- find magnitude, a unit vector and a displacement $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$
- write and use $\mathbf{r} = \mathbf{a} + t\mathbf{b}$; test for intersection
- use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$ to find angles / perpendicularity

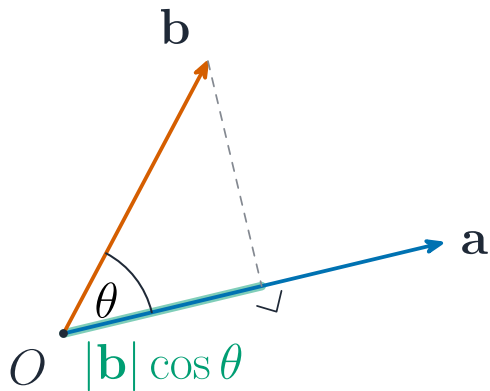
Method — Line and scalar product

Angle between $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$: $\mathbf{a} \cdot \mathbf{b} = 8$, $|\mathbf{a}| = |\mathbf{b}| = 3$, so $\cos \theta = \frac{8}{9}$, $\theta = 27.3^\circ$.



A line through a point with the direction vector added repeatedly

Method — Line and scalar product



Two vectors with the angle between them and a projection

Practice 2.1 ■ 2 marks

Find $|\mathbf{v}|$ for $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$.

Solution 2.1

Worked steps

$$|\mathbf{v}| = \sqrt{9 + 16 + 144} = \sqrt{169} = 13 \quad \text{M1} \quad \text{A1}.$$

Practice 2.2 ■ 2 marks

Find $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

Solution 2.2

Method: Line and scalar product

Worked steps

$$\mathbf{a} \cdot \mathbf{b} = 2 - 3 + 1 = 0 \quad \text{M1} \quad \text{A1}.$$

Exam-style 3.1 ■ 1 mark

Two vectors are perpendicular when their scalar product is:

A 1

B 0

C the product of their magnitudes

D undefined

Solution 3.1

Method: Line and scalar product

A 1

B 0



C the product of their magnitudes

D undefined

Worked steps

B. Perpendicular vectors have scalar product 0.

Exam-style 3.2 ■ 2 marks

The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(a) Find the position vector of the point on l where $t = 3$.

(b) Find the acute angle between l and the direction $\mathbf{d} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$.

Solution 3.2

Method: Line and scalar product

Worked steps

$$(a) \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ -1 \end{pmatrix} \quad \text{M1 A1}.$$

$$(b) \text{ direction of } l \text{ is } (2, 1, -1); \mathbf{b} \cdot \mathbf{d} = 2 + 2 - 2 = 2 \quad \text{M1}; |\mathbf{b}| = \sqrt{6}, |\mathbf{d}| = 3 \quad \text{M1};$$
$$\cos \theta = \frac{2}{3\sqrt{6}} = 0.2722 \Rightarrow \theta = 74.2^\circ \quad \text{M1 A1}.$$

Exam-style 3.3 ■ 2 marks

Points $A(1, 2, 1)$, $B(3, 3, 4)$ and $C(2, 0, 3)$ are given.

- (a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$.
- (b) Find angle BAC .

Solution 3.3

Worked steps

$$(a) \vec{AB} = (2, 1, 3), \vec{AC} = (1, -2, 2) \quad \text{M1 A1}.$$

$$(b) \vec{AB} \cdot \vec{AC} = 2 - 2 + 6 = 6; |\vec{AB}| = \sqrt{14}, |\vec{AC}| = 3 \quad \text{M1}; \cos(BAC) = \frac{6}{3\sqrt{14}} = 0.5345 \Rightarrow BAC = 57.7^\circ \quad \text{M1 A1}.$$