# Past-paper walk-through

Integration ▪ 3.5

eXercise

# Questions in this set

- 9709/31 N25 Q1 ▪ 4 marks
- 9709/31 J25 Q9 ▪ 10 marks
- 9709/31 J25 Q11 ▪ 11 marks
- 9709/32 J25 Q10 ▪ 8 marks
- 9709/32 J25 Q11 ▪ 11 marks

# Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

# Question 1

9709/31 N25 ▪ Q1 ▪ 4 marks

Find the exact value of $\int_1^2 \ln 3x \, dx$. Give your answer in the form $a + \ln b$, where $a$ and $b$ are integers. **[4]**

# Solution 1

## Mark scheme

- By parts: $\int \ln 3x \, dx = x \ln 3x - x$. So $\int_1^2 \ln 3x \, dx = (2\ln 6 - 2) - (\ln 3 - 1) = 2\ln 6 - \ln 3 - 1 = \ln 12 - 1 = -1 + \ln 12$.

# Question 9

9709/31 J25 ▪ Q9 ▪ 10 marks

The constant $a$ is such that $\displaystyle\int_1^a 6x\ln x\,dx = 4$.

**(a)** Show that $a = \exp\left(\frac{1}{6}\left(\frac{5}{a^2} + 3\right)\right)$, where $\exp(x)$ denotes $e^x$. **[5]**

**(b)** Verify by calculation that $a$ lies between 2 and 2.1. **[2]**

**(c)** Use an iterative formula based on the equation in part (a) to determine $a$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. **[3]**

# Solution 9

## (a)

**Mark scheme**

- By parts: $\int 6x \ln x \, dx = 3x^2 \ln x - \frac{3}{2}x^2$. Limits 1 to $a$ equal 4: $3a^2 \ln a - \frac{3}{2}a^2 + \frac{3}{2} = 4 \Rightarrow \ln a = \frac{1}{6}\left(\frac{5}{a^2} + 3\right)$, i.e. $a = \exp\left(\frac{1}{6}\left(\frac{5}{a^2} + 3\right)\right)$ (AG).

# Solution 9

## (b)

**Mark scheme**

- With $f(x) = 6x \ln x - 3x^2$: $f(2) = 4.636 < 5$ and $f(2.1) = 6.402 > 5$, so $a$ lies between 2 and 2.1.

# Solution 9

## (c)

**Mark scheme**

- Iterating $a_{n+1} = \exp\left(\frac{1}{6}\left(\frac{5}{a_n^2} + 3\right)\right)$ converges to $a = 2.02$ (2 d.p.).

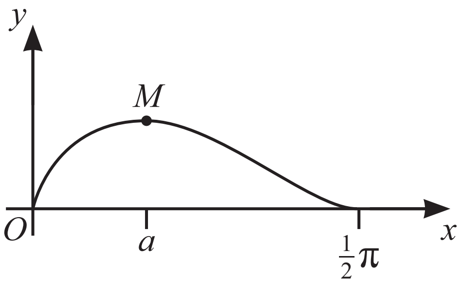
# Question 11

9709/31 J25 ▪ Q11 ▪ 11 marks

The diagram shows the curve $y = \cos x\sqrt{\sin 2x}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curve has a maximum point at $M$, where $x = a$.

**(a)** Find the exact value of $a$. **[6]**

**(b)** The region enclosed between the $x$-axis and the curve is rotated through $2\pi$ radians about the $x$-axis. Find the exact volume of the solid generated. **[5]**



# Solution 11

## (a)

**Mark scheme**

- $\dfrac{dy}{dx} = -\sin x\sqrt{\sin 2x} + \cos x \cdot \dfrac{\cos 2x}{\sqrt{\sin 2x}} = 0 \Rightarrow \cos 3a = 0 \Rightarrow a = \frac{1}{6}\pi.$

# Solution 11

## (b)

**Mark scheme**

- $V = \pi \int_0^{\pi/2} \cos^2 x \sin 2x \, dx = \pi \int_0^{\pi/2} 2\cos^3 x \sin x \, dx = \pi \left[ -\frac{1}{2}\cos^4 x \right]_0^{\pi/2} = \frac{1}{2}\pi.$

# Question 10

9709/32 J25 ▪ Q10 ▪ 8 marks

**(a)** Find the quotient and remainder when $x^2$ is divided by $1 + 4x^2$. **[2]**

**(b)** Find the exact value of $\displaystyle\int_0^{0.5} x\tan^{-1}(2x)\,dx$. **[6]**

# Solution 10

## (a)

**Mark scheme**

- Quotient $\frac{1}{4}$, remainder $-\frac{1}{4}$ (i.e. $x^2 = \frac{1}{4}(1+4x^2) - \frac{1}{4}$).

# Solution 10

**(b)**

**Mark scheme**

- Integrating by parts and using part (a): $\int_0^{0.5} x \tan^{-1}(2x)\, dx = \frac{\pi}{16} - \frac{1}{8}$.

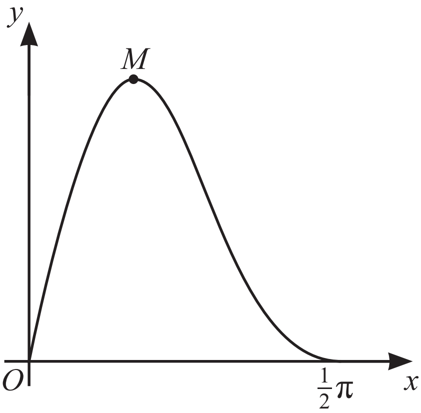
# Question 11

9709/32 J25 ▪ Q11 ▪ 11 marks

The diagram shows the graph of $y = 5\sin 2x\cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its maximum point $M$.

**(a)** Find the exact $x$-coordinate of $M$. **[6]**

**(b)** By using the substitution $u = \cos x$, find the area of the region bounded by the curve, the $x$-axis between $x = 0$ and $x = \frac{1}{4}\pi$, and the line $x = \frac{1}{4}\pi$. **[5]**



# Solution 11

## (a)

**Mark scheme**

- $y = \frac{5}{2}\sin 2x + \frac{5}{4}\sin 4x \Rightarrow \frac{dy}{dx} = 5\cos 2x + 5\cos 4x = 0 \Rightarrow$ $2\cos^2 2x + \cos 2x - 1 = 0 \Rightarrow \cos 2x = \frac{1}{2}$. So $x = \frac{\pi}{6}$.

# Solution 11

## (b)

### Mark scheme

- With $u = \cos x$: area $= \displaystyle\int_0^{\pi/4} 10 \sin x \cos^3 x \, dx = \left[-\tfrac{5}{2}\cos^4 x\right]_0^{\pi/4} = \tfrac{15}{8}$.