Exercise walk-through

Integration ▪ 3.5

eXercise

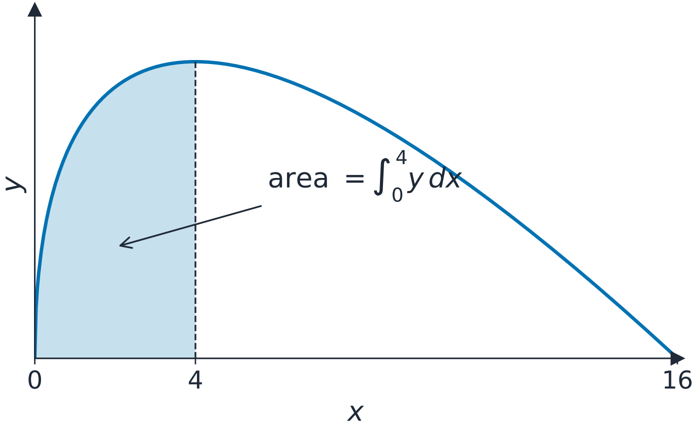
You must be able to

- recognise $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ and $\int \frac{kf'(x)}{f(x)}\,dx = k\ln|f(x)| + C$
- integrate by parts and by a given substitution
- integrate a rational function via partial fractions

Method — By parts & the log pattern

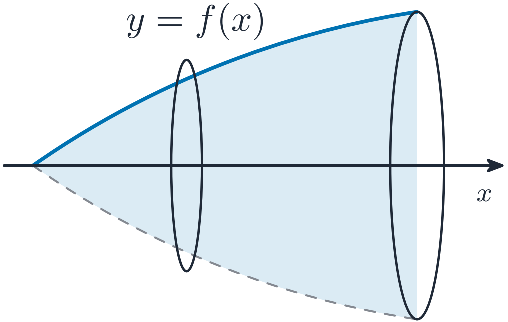
$\int x \cos x \, dx$ — by parts ($u = x$, $dv = \cos x \, dx$):
$= x \sin x - \int \sin x \, dx = x \sin x + \cos x + C$.
$\int \frac{2x}{x^2+1} \, dx = \ln(x^2+1) + C$ (top is the derivative of the bottom).

Method — By parts & the log pattern



A curve with the region under it shaded

Method — By parts & the log pattern



rotate the region around the x-axis to sweep a solid

An application of integration: a volume of revolution

Practice 2.1 ▪ 2 marks

Find $\displaystyle\int \frac{1}{x^2+9}\, dx$.

Solution 2.1

Method: By parts & the log pattern

Worked steps

$\frac{1}{3}\tan^{-1}\dfrac{x}{3} + C$ **M1** **A1**.

Practice 2.2 ▪ 2 marks

Find $\displaystyle\int \frac{3x^2}{x^3+1}\,dx$.

Solution 2.2

Method: By parts & the log pattern

Worked steps

$\ln|x^3+1| + C$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

$\displaystyle\int xe^x\,dx$ equals:

A $xe^x - e^x + C$

B $xe^x + e^x + C$

C $\frac{1}{2}x^2e^x + C$

D $e^x + C$

Solution 3.1

- **A** $xe^x - e^x + C$ ✓
- **B** $xe^x + e^x + C$
- **C** $\frac{1}{2}x^2e^x + C$
- **D** $e^x + C$

Worked steps

A. By parts: $xe^x - \int e^x\,dx = xe^x - e^x + C$.

Exam-style 3.2 ▪ 5 marks

Use the substitution $u = 1 + x^2$ to find $\displaystyle\int_0^2 x\sqrt{1+x^2}\,dx$.

Solution 3.2

Worked steps

$u = 1 + x^2 \Rightarrow du = 2x\,dx$, so $x\,dx = \frac{1}{2}du$ **M1**; limits $x = 0 \to u = 1$, $x = 2 \to u = 5$ **M1**; $\int_1^5 \frac{1}{2}\sqrt{u}\,du = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_1^5 = \frac{1}{3}(5^{3/2} - 1)$ **M1** **M1**; $= \frac{1}{3}(11.180 - 1) = 3.39$ **A1**.

Exam-style 3.3 ▪ 3 marks

(a) Express $\dfrac{4}{(x)(x+2)}$ in partial fractions.

(b) Hence find $\displaystyle\int_1^3 \frac{4}{x(x+2)}\,dx$, giving an exact answer.

Solution 3.3

Method: By parts & the log pattern

Worked steps

(a) $4 = A(x+2) + Bx$; $x = 0 \Rightarrow A = 2$; $x = -2 \Rightarrow B = -2$ **M1** **M1**; $\dfrac{2}{x} - \dfrac{2}{x+2}$ **A1**.

(b) $\displaystyle\int_1^3 \left(\frac{2}{x} - \frac{2}{x+2}\right) dx = \Big[2\ln x - 2\ln(x+2)\Big]_1^3 = 2\ln \frac{x}{x+2}\Big|_1^3$ **M1**;

$= 2\left(\ln \tfrac{3}{5} - \ln \tfrac{1}{3}\right) = 2\ln \tfrac{9}{5}$ **M1** **A1**.