Past-paper walk-through

Differentiation ▪ 3.4

eXercise

Questions in this set

- 9709/31 N25 Q4 ▪ 5 marks
- 9709/31 N25 Q7 ▪ 5 marks
- 9709/31 J25 Q4 ▪ 6 marks
- 9709/31 J25 Q11 ▪ 11 marks
- 9709/32 J25 Q11 ▪ 11 marks

Reading the mark scheme

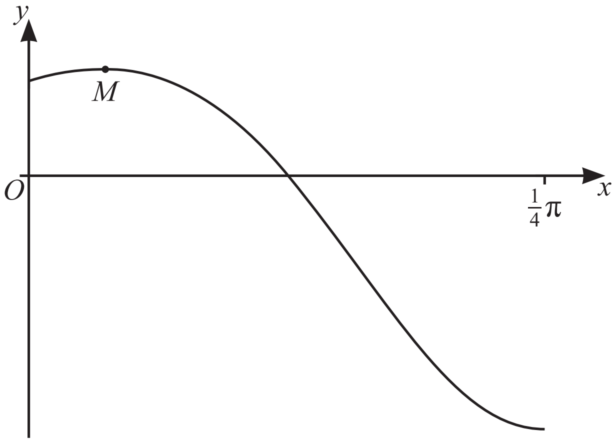
- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

9709/31 N25 ▪ Q4 ▪ 5 marks

The diagram shows the graph of $y = e^{\sin 2x} \cos 4x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$, and its maximum point M. Find the x-coordinate of M. **[5]**

Question 4



Solution 4

Mark scheme

- $\frac{dy}{dx} = e^{\sin 2x}(2\cos 2x \cos 4x - 4\sin 4x) = 0 \Rightarrow \cos 4x = 4\sin 2x \Rightarrow$ $2\sin^2 2x + 4\sin 2x - 1 = 0 \Rightarrow \sin 2x = \frac{\sqrt{6}-2}{2}$. So $x = \frac{1}{2}\sin^{-1}\frac{\sqrt{6}-2}{2} \approx 0.113$.

Question 7

9709/31 N25 ▪ Q7 ▪ 5 marks

The parametric equations of a curve are $x = t^2 - \ln(2t+1)$, $y = \dfrac{t}{2t+1}$. Obtain a simplified expression for $\dfrac{dy}{dx}$ in terms of t. **[5]**

Solution 7

Mark scheme

- $\dfrac{dy}{dt} = \dfrac{1}{(2t+1)^2}$, $\dfrac{dx}{dt} = 2t - \dfrac{2}{2t+1} = \dfrac{2(2t-1)(t+1)}{2t+1}$. So
  $$\frac{dy}{dx} = \frac{1}{2(2t+1)(2t-1)(t+1)} = \frac{1}{2(t+1)(4t^2-1)}.$$

Question 4

9709/31 J25 ▪ Q4 ▪ 6 marks

The parametric equations of a curve are $x = e^{\tan t}$, $y = 3\tan^2 t$. Find the equation of the tangent to the curve at the point $(e, 3)$. Give your answer in the form $y = mx + c$, where m and c are exact. **[6]**

Solution 4

Mark scheme

- At $(e, 3)$: $\tan t = 1$. $\dfrac{dx}{dt} = \sec^2 t\, e^{\tan t}$, $\dfrac{dy}{dt} = 6\tan t \sec^2 t$, so $\dfrac{dy}{dx} = 6\tan t\, e^{-\tan t} = \dfrac{6}{e}$. Tangent: $y = \dfrac{6}{e}x - 3$.

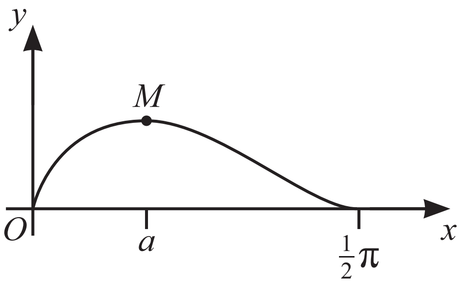
Question 11

9709/31 J25 ▪ Q11 ▪ 11 marks

The diagram shows the curve $y = \cos x\sqrt{\sin 2x}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curve has a maximum point at M, where $x = a$.

(a) Find the exact value of a. **[6]**

(b) The region enclosed between the x-axis and the curve is rotated through 2π radians about the x-axis. Find the exact volume of the solid generated. **[5]**



Solution 11

(a)

Mark scheme

- $\dfrac{dy}{dx} = -\sin x\sqrt{\sin 2x} + \cos x \cdot \dfrac{\cos 2x}{\sqrt{\sin 2x}} = 0 \Rightarrow \cos 3a = 0 \Rightarrow a = \frac{1}{6}\pi.$

Solution 11

(b)

Mark scheme

- $V = \pi \int_0^{\pi/2} \cos^2 x \sin 2x \, dx = \pi \int_0^{\pi/2} 2\cos^3 x \sin x \, dx = \pi \left[-\frac{1}{2}\cos^4 x \right]_0^{\pi/2} = \frac{1}{2}\pi.$

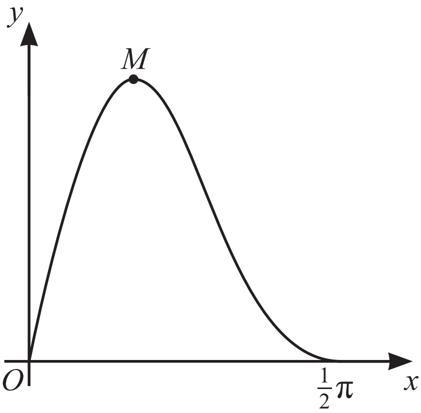
Question 11

9709/32 J25 ▪ Q11 ▪ 11 marks

The diagram shows the graph of $y = 5\sin 2x\cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its maximum point M.

(a) Find the exact x-coordinate of M. **[6]**

(b) By using the substitution $u = \cos x$, find the area of the region bounded by the curve, the x-axis between $x = 0$ and $x = \frac{1}{4}\pi$, and the line $x = \frac{1}{4}\pi$. **[5]**



Solution 11

(a)

Mark scheme

- $y = \frac{5}{2}\sin 2x + \frac{5}{4}\sin 4x \Rightarrow \frac{dy}{dx} = 5\cos 2x + 5\cos 4x = 0 \Rightarrow 2\cos^2 2x + \cos 2x - 1 = 0 \Rightarrow \cos 2x = \frac{1}{2}$. So $x = \frac{\pi}{6}$.

Solution 11

(b)

Mark scheme

- With $u = \cos x$: area $= \displaystyle\int_0^{\pi/4} 10 \sin x \cos^3 x \, dx = \left[-\tfrac{5}{2}\cos^4 x\right]_0^{\pi/4} = \tfrac{15}{8}$.