Exercise walk-through

Differentiation ▪ 3.4

eXercise

You must be able to

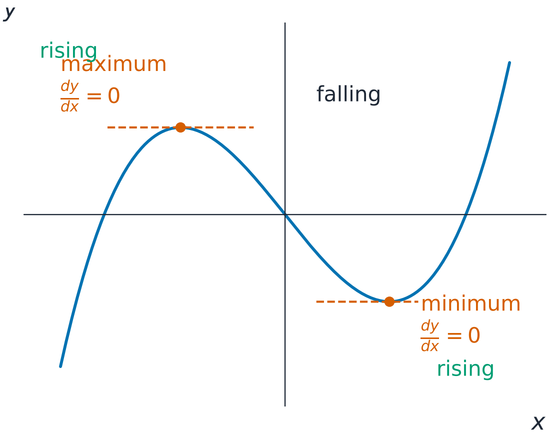
- use $\dfrac{d}{dx}\tan^{-1} x = \dfrac{1}{1+x^2}$
- apply the product, quotient and chain rules
- differentiate parametric and implicit relations

Method — Mixed rules

$y=\tan^{-1}(2x)$: chain rule $\rightarrow \dfrac{dy}{dx}=\dfrac{1}{1+(2x)^2}\cdot 2=\dfrac{2}{1+4x^2}$.

Implicit $e^{2y}=x+y$: $2e^{2y}\dfrac{dy}{dx}=1+\dfrac{dy}{dx}\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{2e^{2y}-1}$.

Method — Mixed rules



A curve with a maximum and a minimum, each with a flat tangent

Practice 2.1 ▪ 2 marks

Differentiate $y = \tan^{-1}(x^2)$.

Solution 2.1

Worked steps

$$\frac{dy}{dx} = \frac{2x}{1 + x^4}$$ **M1** **A1** *chain.*

Practice 2.2 ▪ 2 marks

Differentiate $y = x \ln x$.

Solution 2.2

Worked steps

$$\frac{dy}{dx} = \ln x + 1$$ **M1** **A1** *product.*

Exam-style 3.1 ▪ 1 mark

$\dfrac{d}{dx}\tan^{-1}(3x)$ equals:

A $\dfrac{3}{1+9x^2}$

B $\dfrac{1}{1+3x^2}$

C $\dfrac{3}{1+3x^2}$

D $\dfrac{1}{1+9x^2}$

Solution 3.1

A $\dfrac{3}{1+9x^2}$ ✓

B $\dfrac{1}{1+3x^2}$

C $\dfrac{3}{1+3x^2}$

D $\dfrac{1}{1+9x^2}$

Worked steps

A. $\dfrac{1}{1+(3x)^2} \cdot 3 = \dfrac{3}{1+9x^2}$.

Exam-style 3.2 ▪ 4 marks

A curve has parametric equations $x = 1 + 2\cos t$, $y = 3\sin t$ for $0 \leqslant t \leqslant \pi$. Find $\dfrac{dy}{dx}$ in terms of t and the gradient at $t = \frac{\pi}{6}$.

Solution 3.2

Method: Mixed rules

Worked steps

$\dfrac{dx}{dt} = -2\sin t$, $\dfrac{dy}{dt} = 3\cos t$ **M1**; $\dfrac{dy}{dx} = \dfrac{3\cos t}{-2\sin t} = -\frac{3}{2}\cot t$ **A1**; at $t = \frac{\pi}{6}$:
$-\frac{3}{2}\cot 30^\circ = -\frac{3}{2}\sqrt{3} = -\frac{3\sqrt{3}}{2}$ **M1** **A1**.

Exam-style 3.3 ▪ 3 marks

A curve is given implicitly by $x^2y + y^3 = 10$.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

(b) Find the gradient at the point $(1, 2)$.

Solution 3.3

Method: Mixed rules

Worked steps

(a) $2xy + x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$ **M1**; $\frac{dy}{dx} = -\frac{2xy}{x^2 + 3y^2}$ **M1** **A1**.

(b) at $(1, 2)$: $-\frac{2(1)(2)}{1 + 12} = -\frac{4}{13}$ **M1** **A1**.