Exercise walk-through

Trigonometry ▪ 3.3

eXercise

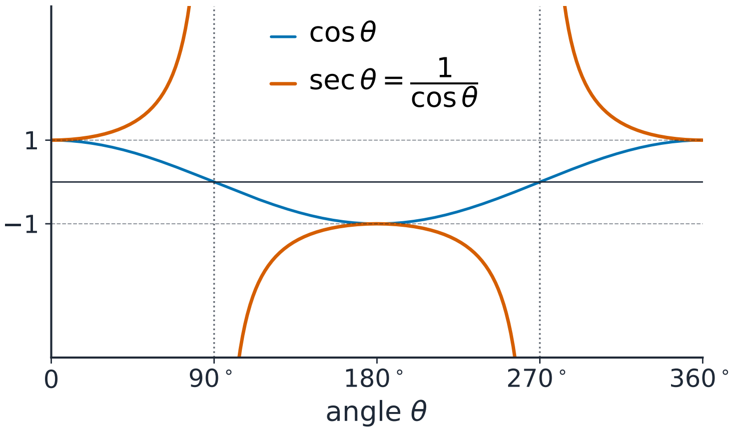
You must be able to

- prove identities using $\sec^2\theta \equiv 1 + \tan^2\theta$ etc.
- expand and use compound/double-angle formulae
- express $a\sin\theta + b\cos\theta$ as $R\sin(\theta \pm \alpha)$ and solve

Method — R-formula in an equation

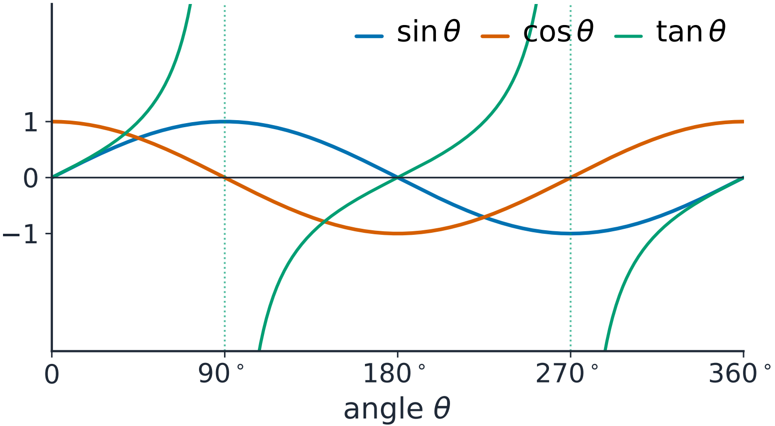
Solve $3\sin\theta + 4\cos\theta = 2$ **for** $0^\circ \leqslant \theta \leqslant 360^\circ$**.** Write LHS as $5\sin(\theta + 53.13^\circ)$: $\sin(\theta + 53.13^\circ) = 0.4$, so $\theta + 53.13^\circ = 23.6^\circ$ or 156.4° (+360°). Taking those in range gives $\theta = 103.3^\circ$ or 330.5°.

Method — R-formula in an equation



The cosine curve and its reciprocal secant

Method — R-formula in an equation



The R-formula rewrites a sum as a single sinusoid

Practice 2.1 ▪ 1 mark

Write $\cos 2\theta$ in terms of $\cos\theta$ only.

Solution 2.1

Method: R-formula in an equation

Worked steps

$\cos 2\theta = 2\cos^2\theta - 1$ **B1**.

Practice 2.2 ▪ 1 mark

Find R for $7\sin\theta + 24\cos\theta = R\sin(\theta + \alpha)$.

Solution 2.2

Worked steps

$R = \sqrt{7^2 + 24^2} = 25$ **B1**.

Exam-style 3.1 ▪ 1 mark

Using $\sin 2\theta = 2\sin\theta\cos\theta$, the value of $\sin 2\theta$ when $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$ is:

A $\frac{12}{25}$

B $\frac{24}{25}$

C $\frac{7}{25}$

D $\frac{6}{5}$

Solution 3.1

A $\frac{12}{25}$

B $\frac{24}{25}$ ✓

C $\frac{7}{25}$

D $\frac{6}{5}$

Worked steps

B. $2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$.

Exam-style 3.2 ▪ 3 marks

(a) Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, with $R > 0$ and $0^\circ < \alpha < 90^\circ$.
(b) Hence solve $4\sin\theta - 3\cos\theta = 2$ for $0^\circ \leqslant \theta \leqslant 360^\circ$.

Solution 3.2

Method: R-formula in an equation

Worked steps

(a) $R = \sqrt{4^2 + 3^2} = 5$ **M1**; $\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.9^\circ$ **M1**; $5\sin(\theta - 36.9^\circ)$ **A1**.
(b) $\sin(\theta - 36.9^\circ) = 0.4 \Rightarrow \theta - 36.9^\circ = 23.6^\circ$ or 156.4° **M1** **M1**; $\theta = 60.5^\circ$ or 193.3° **A1** **A1**.

Exam-style 3.3 ▪ 3 marks

Prove the identity $\dfrac{\sin 2\theta}{1 + \cos 2\theta} \equiv \tan\theta$.

Solution 3.3

Worked steps

$\sin 2\theta = 2\sin\theta\cos\theta$ and $1+\cos 2\theta = 2\cos^2\theta$ M1 M1; ratio $= \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$ A1.