

Exercise walk-through

Logarithmic and exponential functions ■ 3.2

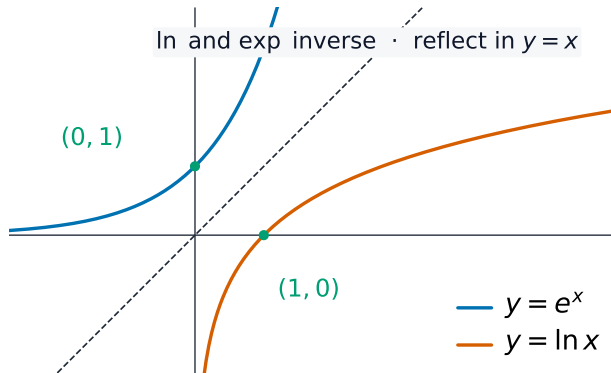
eXercise

You must be able to

- use the laws of logarithms and $e^x/\ln x$ as inverses
- solve equations with the unknown in the power or inside a log
- handle exponential models (growth/decay)

Method — Solving log/exp equations

Solve $e^{2x} - 5e^x + 6 = 0$. Let $u = e^x$: $u^2 - 5u + 6 = 0 \Rightarrow (u - 2)(u - 3) = 0$. So $e^x = 2$ or 3 , giving $x = \ln 2$ or $\ln 3$.



$\hat{e^x}$ and $\ln x$ mirrored across $y = x$

Practice 2.1 ■ 2 marks

Solve $\ln(x - 1) = 2$, giving x to 3 s.f.

Solution 2.1

Method: Solving log/exp equations

Worked steps

$$x - 1 = e^2 \Rightarrow x = 1 + e^2 = 8.39 \quad \text{M1} \quad \text{A1}.$$

Practice 2.2 ■ 1 mark

Write $2 \ln a - \ln b$ as a single logarithm.

Solution 2.2

Worked steps

$$\ln \frac{a^2}{b} \quad \text{B1}.$$

Exam-style 3.1 ■ 1 mark

The solution of $5e^{-x} = 2$ is:

A $\ln 2.5$

B $-\ln 2.5$

C $\ln 0.4$

D $\ln 10$

Solution 3.1

A $\ln 2.5$ **B** $-\ln 2.5$ **C** $\ln 0.4$ **D** $\ln 10$ **Worked steps**

A. $e^{-x} = 0.4 \Rightarrow -x = \ln 0.4 \Rightarrow x = -\ln 0.4 = \ln 2.5.$

Exam-style 3.2 ■ 4 marks

Solve the equation $e^{2x} - 7e^x + 10 = 0$, giving exact answers.

Solution 3.2

Method: Solving log/exp equations

Worked steps

let $u = e^x$: $u^2 - 7u + 10 = 0 \Rightarrow (u - 2)(u - 5) = 0$ **M1** **M1**; $e^x = 2$ or 5 **A1**;
 $x = \ln 2$ or $\ln 5$ **A1**.

Exam-style 3.3 ■ 3 marks

A quantity decays so that $P = P_0 e^{-kt}$. After 5 years P has fallen to half of P_0 .

- (a) Find k , to 3 s.f.
- (b) Find the time for P to fall to one tenth of P_0 .

Solution 3.3

Worked steps

$$(a) \frac{1}{2}P_0 = P_0 e^{-5k} \Rightarrow e^{-5k} = 0.5 \Rightarrow -5k = \ln 0.5 \quad \text{M1 M1}; k = 0.139 \quad \text{A1}.$$

$$(b) 0.1 = e^{-kt} \Rightarrow t = \frac{\ln 0.1}{-0.139} = 16.6 \text{ years} \quad \text{M1 A1}.$$